Model description of AGCM5 of GFD-Dennou-Club edition

SWAMP project, GFD-Dennou-Club

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AGCM5 of the GFD-DENNOU CLUB edition is a three-dimensional primitive system on a sphere (Swamp Project, 1998). The model is basically the same as that used in Numaguti (1993), and its details are described in Numaguti(1992). In the following subsections, we briefly describe the model system and its implementation.

1 Basic equations

The dynamical part of the model consists of the vorticity, divergence, hydrostatic, humidity and thermodynamic equations as follows.

$$
\begin{aligned}
\frac{d\zeta}{dt} &= \frac{1}{a\cos\varphi}\frac{\partial}{\partial\lambda}\{-(\zeta+f)u-\dot{\sigma}\frac{\partial u}{\partial\sigma}-\frac{RT'}{a\cos\varphi}\frac{\partial\pi}{\partial\lambda}\} \\
&\quad -\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}[\{(\zeta+f)v-\dot{\sigma}\frac{\partial u}{\partial\sigma}-\frac{RT'}{a}\frac{\partial\pi}{\partial\varphi}\}\cos\varphi]-F_{\zeta}^{diff}, && (1)\\
\frac{dD}{dt} &= \frac{1}{a\cos\varphi}\frac{\partial}{\partial\lambda}\{(\zeta+f)v-\dot{\sigma}\frac{\partial u}{\partial\sigma}-x\frac{RT'}{a}\frac{\partial\pi}{\partial\varphi}\} \\
&\quad +\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}[\{(\zeta+f)v-\dot{\sigma}\frac{\partial u}{\partial\sigma}-\frac{RT'}{a}\frac{\partial\pi}{\partial\varphi}\}\cos\varphi] \\
&\quad -\nabla^2(\Phi+R\overline{T}\pi+E)-F_D^{diff}, && (2)\\
\frac{d\pi}{dt} &= -\frac{1}{a\cos\varphi}\frac{\partial u}{\partial\lambda}-\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(v\cos\varphi)-\frac{\partial\dot{\sigma}}{\partial\sigma}, && (3)\\
\frac{\partial\Phi}{\partial\sigma} &= -\frac{RT}{\sigma}, && (4)\\
\frac{dq}{dt} &= \frac{g}{p_s}\frac{\partial F_q^{vdf}}{\partial\sigma}+F_q^{vdf}+S_q^{cond}, && (5)\\
\frac{dT}{dt} &= \frac{RT}{c_p}\left\{\frac{\partial\pi}{\partial t}+\frac{u}{a\cos\varphi}\frac{\partial\pi}{\partial\lambda}+\frac{v}{a}\frac{\partial\pi}{\partial\varphi}+\frac{\dot{\sigma}}{\sigma}\right\} \\
&\quad +\frac{1}{c_p}\left(\frac{g}{p_s}\frac{\partial F_T^{vdf}}{\partial\sigma}+\frac{g}{p_s}\frac{\partial F_{rad}^{vdf}}{\partial\sigma}\right)+F_T^{diff}+LS_q^{cond}. && (6)
\end{aligned}
$$

In these equations,

$$
\begin{aligned}
\frac{d}{dt} &\equiv \frac{\partial}{\partial t}+\frac{u}{a\cos\varphi}\frac{\partial}{\partial\lambda}+\frac{v}{a}\frac{\partial}{\partial\varphi}+\dot{\sigma}\frac{\partial}{\partial\sigma}, && (7)\\
\zeta &\equiv \frac{1}{a\cos\varphi}\frac{\partial v}{\partial\lambda}-\frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(u\cos\varphi), && (8)
\end{aligned}
$$

$$
\begin{aligned}
D &\equiv \frac{1}{a\cos\varphi}\frac{\partial u}{\partial\lambda} + \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(v\cos\varphi), && (9)\\
\Phi &\equiv gz, && (10)\\
\pi &\equiv \ln p_s, && (11)
\end{aligned}
$$

where, (λ, φ) are the latitudinal and longitudinal coordinates; σ is the vertical coordinate; u, v are the horizontal wind components; $\dot{\sigma}$ is the vertical wind in σ coordinate; T is temperature; q is specific humidity; p_s is surface pressure; T_g is surface temperature; Φ is geopotential; f is the Coriolis parameter; a is the radius of the earth; g is the acceleration of gravity; R is the gas constant of dry air; c_p is the atmospheric specific heat at constant pressure; L is the latent heat of water; F_ζ^{diff}, F_D^{diff}, F_T^{diff} F_q^{diff} are the horizontal diffusion terms; F_u^{vdf}, F_v^{vdf}, F_T^{vdf}, F_q^{vdf} are the vertical diffusion terms; and S_q^{cond} is the condensation source (loss) of the specific humidity.

The horizontal diffusion terms are represented by a hyper-viscosity type formula,

$$F^{diff} = -(-1)^n K \nabla^{2n} \tag{12}$$

for which $n = 8$ is adopted. The value of K is determined such that the e-folding time of the component with wavenumber 42 is four hours.

2 Discretization

The equations are discretized horizontally by a non-alias latitude-longitude grid and are integrated by the pseudo-spectral method with a triangular truncation. The scheme of Arakawa and Suarez (1983) is utilized for discretizing in the vertical direction. The time integration is performed by the use of a leap-frog scheme for the terms of dynamical processes other than dissipative processes, a backward scheme for the terms of physical processes, and an Euler scheme for the horizontal diffusion terms. The time filter of Asselin (1972) is used to remove the computational mode caused by the leap-frog scheme.

3 Physical processes

The model contains the following simplified physical processes.

3.1 Cumulus parameterization

The cumulus parameterization schemes implemented are the adjustment scheme (Manabe *et al.*, 1965) and Kuo scheme (Kuo, 1974). In the followings, the description of adjustment schmes is given. Condensed water is removed from the system immediately after condensation and hence there are neither rain drops nor clouds.

Moist convective adjustment occurs when the following conditions are satisfied:

- In two neighboring levels, the atmosphere becomes moist unstable, that is , the temperature lapse rate exceeds the moist adiabatic lapse rate.
- Both of the neighboring levels are saturated or oversaturated.

In the process of adjustment, the temperature and specific humidity of the two neighboring levels are instantaneously adjusted to values so that the levels are saturated and the temperature lapse rate between them coincides with the moist adiabatic lapse rate. In due course, moist static energy has to be conserved:

$$\sum_{l=k-1}^{k} (C_p T_l + L q_l)\Delta p_l = \sum_{l=k-1}^{k} (C_p \hat{T}_l + L \hat{q}_l)\Delta p_l, \tag{13}$$

where k is the index of the vertical level. Here, $\hat{T}, \hat{q}$ denote the values before adjustment, while T, q denote the values after the adjustment.

The moist unstable condition for layer $(k, k-1)$ is evaluated by

$$\begin{aligned} ST_{k-1/2} &\equiv \hat{T}_k - \hat{T}_{k-1} + \frac{L}{C_p}\left(q^*(\hat{T}_k, p_k) - q^*(\hat{T}_{k-1}, p_{k-1})\right) - \frac{R}{C_p}\frac{\Delta p_{k-1/2}}{p_{k-1/2}}\frac{\hat{T}_k + \hat{T}_{k-1}}{2} \\ &> 0. \end{aligned} \tag{14}$$

The saturation condition of two neighboring levels is simply expressed by

$$\hat{q}_{k-1} > q^*(\hat{T}_{k-1}, p_{k-1}), \quad \hat{q}_k > q^*(\hat{T}_k, p_k). \tag{15}$$

The adjusted values (T_k, q_k) are determined as

$$T_{k-1} = \hat{T}_{k-1} + \Delta T_{k-1}, \tag{16}$$

$$T_k = \hat{T}_k + \Delta T_k, \tag{17}$$

$$q_{k-1} = q^*(\hat{T}_{k-1}, p_{k-1}) + \left.\frac{\partial q^*}{\partial T}\right|_{k-1} \Delta T_{k-1}, \tag{18}$$

$$q_k = q^*(\hat{T}_k, p_k) + \left.\frac{\partial q^*}{\partial T}\right|_{k} \Delta T_k, \tag{19}$$

$$\Delta T_k = \frac{(1+\gamma_{k-1})\Delta p_{k-1} S_{k-1/2} + [1 + \gamma_{k-1} - \kappa \frac{\Delta p_{k-1/2}}{2p_{k-1/2}}] \frac{L}{C_p} \Delta \hat{Q}}{(1+\gamma_{k-1})(1+\gamma_k)(\Delta p_{k-1} + \Delta p_k) + \kappa \frac{\Delta p_{k-1/2}}{2p_{k-1/2}} [(1+\gamma_{k-1})\Delta p_{k-1} - (1+\gamma_k)\Delta p_k]}, \tag{20}$$

$$\Delta T_{k-1} = -\frac{(1+\gamma_k)\Delta p_k}{(1+\gamma_{k-1})\Delta p_{k-1}} \Delta T_k + \frac{1}{(1+\gamma_{k-1})\Delta p_{k-1}} \frac{L}{C_p} \Delta T_k, \tag{21}$$

where

$$\gamma_k = \frac{L}{C_p} \left.\frac{\partial q^*}{\partial T}\right|_k, \quad \gamma_{k-1} = \frac{L}{C_p} \left.\frac{\partial q^*}{\partial T}\right|_{k-1}, \tag{22}$$

$$\Delta \hat{Q} = (\hat{q}_{k-1} - q^*(\hat{T}_{k-1}, p_{k-1}))\Delta p_{k-1} + (\hat{q}_k - q^*(\hat{T}_k, p_k))\Delta p_k. \tag{23}$$

Precipitation of one column is calculated as

$$P = \sum_{k=1}^{K} (\hat{q}_k - q_k) \frac{\Delta p_k}{g}. \tag{24}$$

These adjustment procedures are applied to all levels until the vertical temperature and moisture profiles converge.

3.2 Large scale condensation

The large scale condensation scheme is applied at each grid point when the value of the specific humidity exceeds its saturation value. The large scale condensation scheme demands that the value of the specific humidity be adjusted to the saturation value. Condensed water is assumed to fall out as rain and is removed from the system immediately. There are no rain drops or clouds, as in the case of the convective adjustment scheme. In the time integration procedure, the large scale condensation scheme is applied just after the convective adjustment scheme.

The condition for large scale condensation can be denoted by

$$\hat{q} > q^*(\hat{T}, p), \tag{25}$$

where $\hat{T}$, $\hat{q}$ are the values of temperature and specific humidity before the adjustment. After the large scale condensation adjustment, the values of the temperature and specific humidity become

$$q = q^*(T, p) \tag{26}$$

and

$$C_p T + Lq = C_p \hat{T} + L\hat{q}. \tag{27}$$

The solutions to these are obtained by Newton method;

$$T = \hat{T} + \frac{L}{C_p} \frac{\hat{q} - q^*(\hat{T}, p)}{1 + \frac{L}{C_p}\frac{\partial q^*}{\partial T}}, \tag{28}$$

$$q = \hat{q} - \frac{L}{C_p}\frac{\partial q^*}{\partial T} \frac{\hat{q} - q^*(\hat{T}, p)}{1 + \frac{L}{C_p}\frac{\partial q^*}{\partial T}}. \tag{29}$$

The amount of precipitation is calculated by

$$P = \frac{\Delta p}{g}(\hat{q} - q). \tag{30}$$

3.3 Radiation

Radiative flux is calclated by band model. The effects of scattering of solar radiation is not considered. It is assumed that there are no cloud nor rain drops.

As for longwave radiation, we include only the absorption and emission of water and dry air; the effect of scattering is not included. The absorption coefficients of the longwave radiation are chosen so that the cooling profile of the atmosphere roughly resembles the observed one.

Radiation flux $F_{R,i}$ is

$$F_R(z) = (\pi B(T_g) - \pi B(T_s))\, \mathcal{T}^f(z, 0) + \pi B(T(z_T))\mathcal{T}^f(z, z_T) - \int_0^{z_T} \frac{d\pi B}{d\xi}\mathcal{T}^f(z, \xi) d\xi, \tag{31}$$

where $\mathcal{T}^f(z_1, z_2)$ is the flux transfer function between $z = z_1, z_2$ and $\pi B \equiv \sigma_{SB} T^4$ is the source function.

The flux transfer function $\mathcal{T}^f(z_1, z_2)$ is assumed to be given as

$$\mathcal{T}^f(z_1, z_2) = \sum_{i=1}^{N_R} b_i \exp\left(-\delta_R |\tau_{R,i}(z_1) - \tau_{R,i}(z_2)|\right). \tag{32}$$

Here, $\tau_i(z)$ is the optical depth whose value at the top of the atmosphere is 0 and

$$\tau_{R,i}(z) = \int_z^\infty k_{R,i}\rho q dz + \int_z^\infty \bar{k}_{R,i}\rho dz, \tag{33}$$

where $k_{R,i}$ and $\bar{k}_{R,i}$ are the absorption coefficients of water vapor and dry air for wavenumber band i, respectively. The values of these absorption coefficients are assumed to be constant. b_i is the ratio of the energy for wavenumber band i to the total radiative energy and is also assumed to be constant. The default value for δ_R is 1.5. In most cases, N_R is set to be 4, three representing water vapor bands and one representing dry air band. The default values of $k_{R,i}$, $\bar{k}_{R,i}$ and b_i are summarized in Table 1.

Table 1: The values of constants used in the longwave radiation scheme.

Band Number	1	2	3	4
$k_{R,i}$	8.0	1.0	0.1	0.0
$\bar{k}_{R,i}$	0.0	0.0	0.0	5.0×10^{-5}
b_i	0.2	0.1	0.1	0.6

3.4 Vertical mixing

Vertical turbulent mixing is represented by the level II scheme of Mellor and Yamada (1974).

In this scheme of Mellor and Yamada (1974), diffusion coefficients K_M, K_T, and K_q depends on stability and are given as functions of u, v, and T:

$$K_M = l^2 \frac{\Delta|\boldsymbol{v}|}{\Delta z} S_M, \tag{34}$$

$$K_T = K_q = l^2 \frac{\Delta|\boldsymbol{v}|}{\Delta z} S_H. \tag{35}$$

Following Blakadar(1962), the mixing length l is evaluated by

$$l = \frac{\kappa z}{1 + \kappa z / l_0}, \tag{36}$$

where $l_0 = 300$ m and $\kappa = 0.4$ is the Kárman constant.

In these expressions, S_M and S_H are given as functions of the Richardson number.

$$\begin{aligned} S_M &= B_1^{1/2}(1-R_f)^{1/2}\widetilde{S_M}^{3/2}, && (37) \\ S_H &= B_1^{1/2}(1-R_f)^{1/2}\widetilde{S_M}^{1/2}\widetilde{S_H}, && (38) \end{aligned}$$

where

$$\widetilde{S_H} = \frac{\alpha_1 - \alpha_2 R_f}{1-R_f}, \tag{39}$$

$$\widetilde{S_M} = \frac{\beta_1 - \beta_2 R_f}{\beta_3 - \beta_4 R_f}\widetilde{S_H}. \tag{40}$$

R_f is the flux Richardson number, given as

$$R_f = \frac{1}{2\beta_2}\left[\beta_1 + \beta_4 R_{iB} - \sqrt{(\beta_1 + \beta_4 R_{iB})^2 - 4\beta_2\beta_3 R_{iB}}\right], \tag{41}$$

where R_{iB} is the bulk Richardson number and

$$\begin{aligned} \alpha_1 &= 3A_2\gamma_1, && (42) \\ \alpha_2 &= 3A_2(\gamma_1+\gamma_2), && (43) \\ \beta_1 &= A_1B_1(\gamma_1 - C_1), && (44) \\ \beta_2 &= A_1[B_1(\gamma_1 - C_1) + 6A_1 + 3A_2], && (45) \\ \beta_3 &= A_2B_1, && (46) \\ \beta_4 &= A_2[B_1(\gamma_1+\gamma_2) - 3A_1], && (47) \end{aligned}$$

$$(A_1, B_1, A_2, B_2, C_1) = (0.92, 16.6, 0.74, 10.1, 0.08), \tag{48}$$

$$\gamma_1 = \frac{1}{3} - \frac{2A_1}{B_1}, \ \ \gamma_2 = \frac{B_2}{B_1} + 6\frac{A_1}{B_1}. \tag{49}$$

R_{iB} is defined as

$$R_{iB} = \frac{\dfrac{q}{\theta_s}\dfrac{\Delta\theta}{\Delta z}}{\left(\dfrac{\Delta u}{\Delta z}\right)^2 + \left(\dfrac{\Delta v}{\Delta z}\right)^2}. \tag{50}$$

Here, the value of $(\Delta\bullet)$ at the vertical level $k-1/2$ is evaluated as $\bullet_k - \bullet_{k-1}$.

3.5 Surface flux

The surface fluxes of momentum, heat and water vapor are evaluated by the bulk formulae

$$
\begin{aligned}
(F_\lambda)_{1/2} &= -\rho_s C_M |\boldsymbol{v}_1| u_1, & (51)\\
(F_\varphi)_{1/2} &= -\rho_s C_M |\boldsymbol{v}_1| u_1, & (52)\\
(F_T)_{1/2} &= \rho_s C_p C_H |\boldsymbol{v}_1| (T_g - (\frac{T}{\Pi_1})), & (53)\\
(F_q)_{1/2} &= \rho_s \beta C_E |\boldsymbol{v}_1| (q^*(T_g, p_s) - q_1), & (54)
\end{aligned}
$$

where the suffix 1/2 indicates that these are the value at the vertical level 1/2.

Following Louis *et al.* (1982), the bulk coefficients C_M, C_H, and C_E are determined by

$$
C_M = \begin{cases} C_{M0} \left(1 + 10 R_{iB} / \sqrt{1 + 5 R_{iB}}\right)^{-1} & R_{iB} \geq 0, \\ C_{M0} \left[1 - 10 R_{iB} \left(1 + 75 C_{M0} \sqrt{\frac{z_1}{z_{M0}}} |R_{iB}|\right)^{-1}\right] & R_{iB} < 0, \end{cases} \quad (55)
$$

$$
C_H = C_E = \begin{cases} C_{H0} \left(1 + 15 R_{iB} / \sqrt{1 + 5 R_{iB}}\right)^{-1} & R_{iB} \geq 0, \\ C_{H0} \left[1 - 15 R_{iB} \left(1 + 75 C_{H0} \sqrt{\frac{z_1}{z_{M0}}} |R_{iB}|\right)^{-1}\right] & R_{iB} < 0, \end{cases} \quad (56)
$$

where R_{iB} is the bulk Richardson number evaluated by

$$
R_{iB} = \frac{\frac{g}{\theta_s}(\theta_1 - \theta_g)/g}{(u_1/z_1)^2} = \frac{g}{\theta_s} \frac{T_1 (p_s/p_1)^\kappa - T_g}{u_1^2 / z_1}. \quad (57)
$$

C_{M0} and C_{H0} are the bulk coefficients for neutral case:

$$
\begin{aligned}
C_{M0} &= \left(\frac{\kappa}{\ln\left(\frac{z_1}{z_{M0}}\right)}\right), & (58)\\
C_{H0} &= \left(\frac{\kappa}{\ln\left(\frac{z_1}{z_{H0}}\right)}\right), & (59)
\end{aligned}
$$

where z_{M0} and z_{H0} are the roughness parameters of the surface. The values are 10^{-4}m.

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