$$\frac{\partial}{\partial t}\nabla^2\phi + U\frac{\partial}{\partial x}\nabla^2\phi + \left(\beta - \frac{d^2U}{dy^2}\right)\frac{\partial\phi}{\partial x} = 0$$

where $u' \equiv -\dfrac{\partial\phi}{\partial y}, \quad v' \equiv \dfrac{\partial\phi}{\partial x}$