$$\int -2ic_i \frac{|v'|^2}{k|U-c|^2}\frac{dQ}{dy}dy = -2i\frac{c_i}{k}\int \frac{|v'|^2}{(U-c_r)^2+c_i^2}\frac{dQ}{dy}dy$$

$$= -2i\frac{\pi Q'(y_c)}{kU'(y_c)}\mathrm{sgn}(c_i)$$