$$\frac{\partial q'}{\partial t} + U\frac{\partial q'}{\partial x} + v'\frac{dQ}{dy} = 0$$

where $q' = \dfrac{\partial v'}{\partial x} - \dfrac{\partial u'}{\partial y} = \nabla^2\phi,$

$$\frac{dQ}{dy} = \frac{d}{dy}\left(f - \frac{dU}{dy}\right) = \beta - \frac{d^2U}{dy^2}$$