

# 二重拡散対流

— 実験室から惑星まで —

野口 尚史

京都大学大学院 工学研究科 航空宇宙工学専攻

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Mathematical  
Fluid  
Dynamics  
Laboratory

## 導入

二重拡散対流とは

## 基本メカニズム

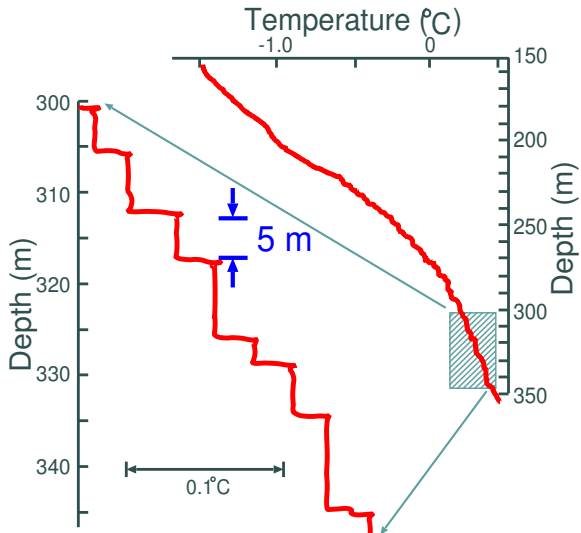
線形安定性

## フィンガー対流

## 拡散型対流

## 二重拡散対流とはどういうものか

北極海での鉛直温度分布



[Neal *et al.*, 1969]

## 二重拡散対流とはどういうものか

冷たい真水の上に温かい塩水をのせる

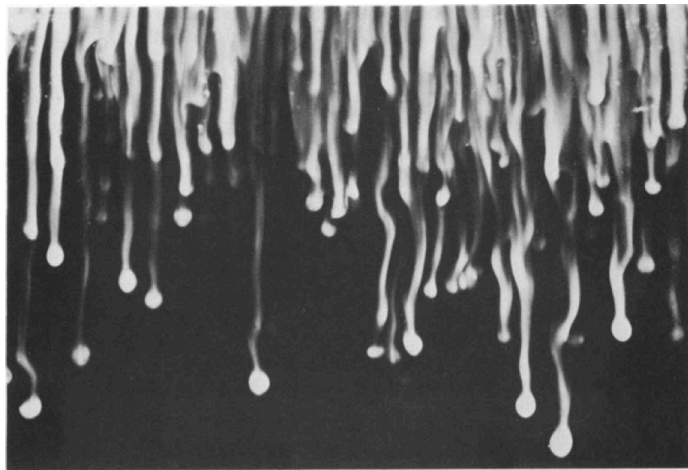


FIGURE 1. A field of salt fingers formed by setting up a stable temperature gradient and pouring a little salt solution on top. The downward-moving fingers were made visible by adding fluorescein to the salt and lighting through a slit from below.

[Huppert&Turner, 1981]

# 二重拡散対流とはどういうものか

## 二重拡散対流が起こる状況

- ▶ 2種類 (あるいは多種の) 成分が密度に寄与
- ▶ 分子拡散の速度が異なる
  - ▶ 熱:塩分 = 100:1
  - ▶ 塩分:砂糖分 = 3:1
- ▶ 成分ごとの鉛直密度勾配が逆向き
- ▶ 正味の密度勾配は下向き
  - ▶ 静的 (かつ非拡散的) に安定

## 二重拡散対流とはどういうものか

### 色分けの規則

warm water / cold water

salty water / fresh water

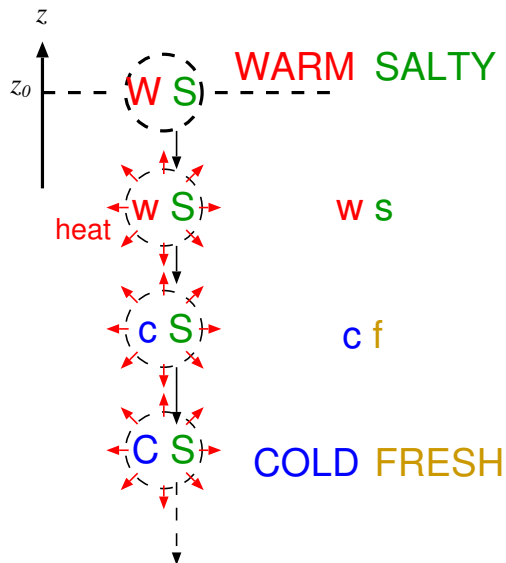
heavy/light

- ▶ 赤っぽい方が軽い
- ▶ 青っぽい方が重い

### 表記の規則

- ▶ 二重拡散対流を引き起こすのは熱と塩分に限らない
- ▶ 拡散する2つの成分(溶質)の濃度は 拡散が速いほうを  $T$ 、遅いほうを  $S$  と表す
- ▶ 線形の状態変化を考える場合がほとんどなので、密度変化への寄与 ( $\alpha(T - T_0), \beta(S - S_0)$ ) を略して  $T, S$  と表す場合も

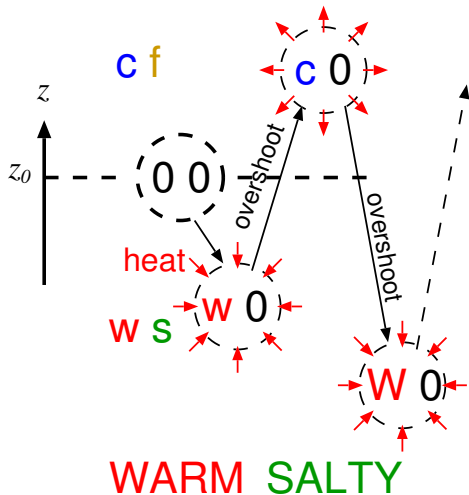
# パーセル法: フィンガー対流



- ▶ 上ほど 高温高塩
- ▶ 下ほど 低温低塩
- ▶ 正味の密度は下ほど大きい (静的安定)
- ▶ 低緯度海洋 (日射による加熱・蒸発)

# パーセル法: 拡散対流

COLD FRESH



- ▶ 上ほど 低温低塩
- ▶ 下ほど 高温高塩
- ▶ 正味の密度は下ほど大きい(静的安定)
- ▶ 高緯度海洋 (大気による冷却・雪氷からの淡水)



## 二重拡散対流の(再)発見

Stommel, Arons & Blanchard (1956): 'Perpetual salt fountain'

**An oceanographical curiosity : the perpetual salt fountain**

In many tropical and subtropical regions of the ocean the warm surface layers have a salinity exceeding that found in the colder waters of polar origin below. If a long tube were lowered from the surface to depth of low salinity water, and the deep water were slowly pumped to the surface through the tube, and the pump then disconnected, the water would continue to flow, by itself, forever. This remarkable phenomenon occurs because slow motion through the tube allows the water inside to attain the same temperature as the surrounding water. Its salinity, and hence density, is therefore less than that of its surroundings outside the tube, and hence the entire column of water inside the tube is buoyant with respect to the fluid outside at the same level. If the direction of pumping is reversed, so that the fluid initially goes downward, it will of course continue to flow downward forever on account of its excess in density over that of the water outside the tube.

Although the attempt has not been made it is likely that in the Central North Atlantic with a tube 2,000 meters long, one might develop a pressure head of as much as two meters at the surface. A simple experiment on a laboratory scale will demonstrate the principle, however. A vertical glass tube, about three inches in diameter (A in Fig. 1) is filled halfway with hot fresh water. Then the same quantity of slightly coloured cold fresh water is introduced through a hole in the bottom (B) care being taken not to produce too much mixing at the interface between the two fluids. An

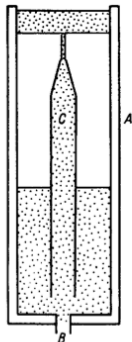
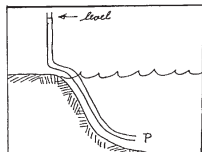


Fig. 1. Laboratory experiment to demonstrate the salt fountain, showing outside glass tube A, with hole in the bottom for filling B, and inner glass tube C to produce the fountain. The shading indicates the coloured water after the fountain has been in operation for some time.

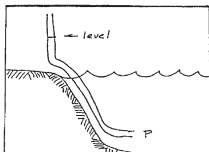
# 二重拡散対流の(再)発見

Stommel, Arons & Blanchard (1956): 'Perpetual salt fountain'

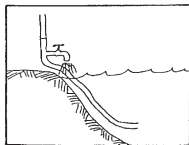
The Salt Fountain



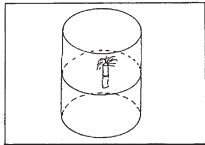
Arnold Arons and Henry Stommel are trying to sketch some kind of hydraulic method using submarine tubes to measure the pressure at some deep ocean point P when the end of the tube is placed at the other end on shore. They recognize that if they fill the tube with fresh water it will quickly equilibrate in temperature with the ocean and on the shore end will stand quite high above sea level.



Henry says that fresh water is dear on Bermuda so why not use the ocean water at the deep end by sucking it into the tube. It is fresher and will still stand above sea-level. They calculate it at about one meter.



Arnold sketches a faucet in the tube at about a half meter above sea-level because he sees that it can run forever.



They both run downstairs to Duncan Blanchard's lab where they build a little working salt fountain in a battery jar.

実際の思い付きの過程

('Exciting Ten Minutes')

- ▶ 当初は深いところの圧力を測る方法を考えていた
- ▶ 深海の水(低塩)で管を満たす
- ▶ まわりの温度になじむ
- ▶ 管内の水位は海面より1m高くなる
- ▶ 穴を開けたらいつまでも流出?
- ▶ 実験室に直行 → 成功

[Ruddick&Gargett, 2003]

## 二重拡散対流の(本当の)発見

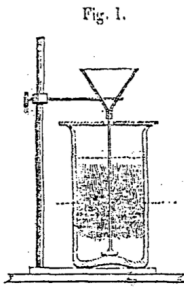
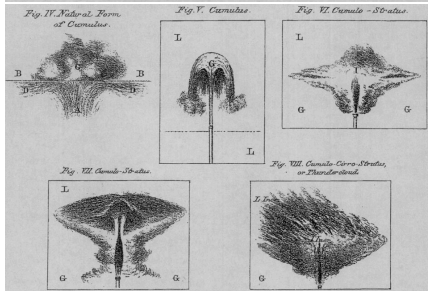
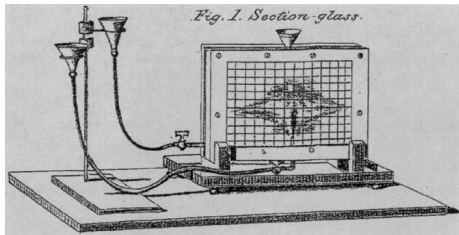


[Schmitt, 1995]

William Stanley Jevons  
(1835–1882)

- ▶ イギリスの経済学者 (!)
- ▶ 「限界効用」の概念
- ▶ ‘景気変動と太陽の黒点の関係’
- ▶ 数論や幾何学にも興味

# 二重拡散対流の(本当の)発見

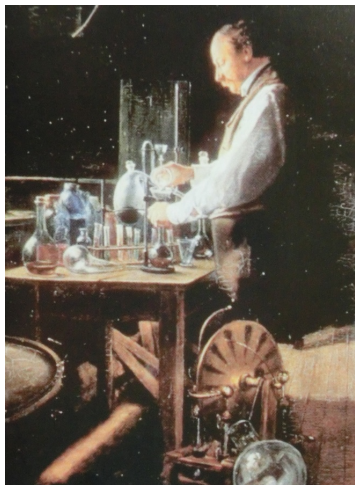


Jevons は雲の生成メカニズムに興味をもち、**温かい砂糖水**を用いて実験をした

- ▶ フィラメント状の構造を見出す
- ▶ メカニズムは解明できず

'On the Cirrous Form of Cloud'  
(1857)

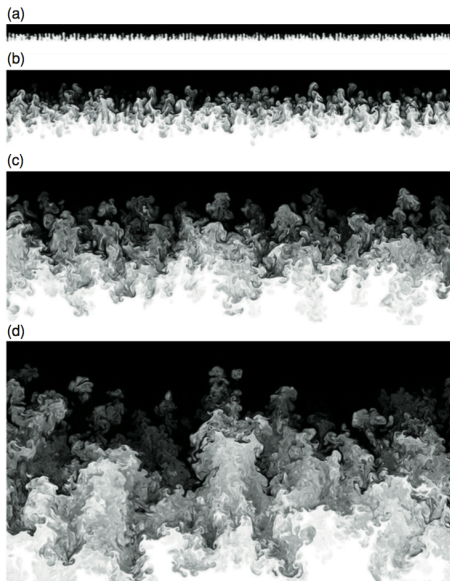
## 二重拡散対流の(本当の)発見



[IOP]

- ▶ Jevons の研究に目を止めた Rayleigh は自分でも実験を試みた
- ▶ 安定性理論を展開したが、拡散を考慮しなかった (1883)
- ▶ 上層が下層より重いために起こると誤って解釈
- ▶ しかし、この理論の成果は **Rayleigh-Taylor 不安定** として残る

# Rayleigh-Taylor 不安定



一見 フィンガー対流と類似  
しかし

- ▶ 発達すると水平スケールが大きくなる
  - ▶ フィンガーでは幅はほぼ一定
- ▶ 発達が非常に速い
  - ▶ R-T: はじめから浮力を持っている
  - ▶ 二重拡散: 拡散ではじめて浮力を得る
- ▶ 全体が乱流状
  - ▶ 本質的に非粘性の現象
  - ▶ ‘自由落下’

[Rathkopf, 2004]

## 2層の境界面 (不連続成層の場合)

finger interface

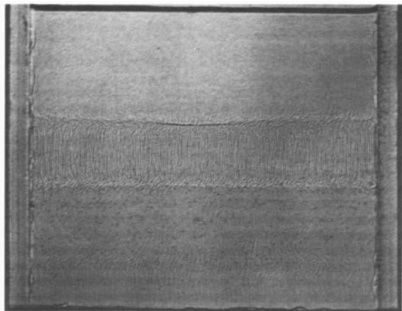


Figure 2 Shadowgraph of a thickened "finger" interface formed by placing a layer of sucrose solution on top of a denser NaCl solution and leaving it for about three hours (cf. Figure 1).

[Turner, 1985]

diffusive interface

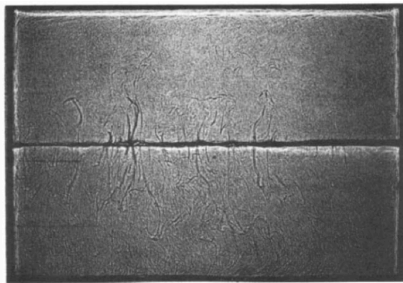
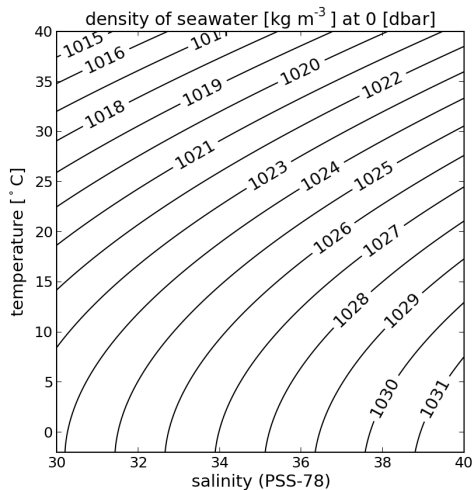


Figure 1 Shadowgraph picture of a "diffusive" interface formed by pouring a layer of NaCl solution ( $\kappa = 1.5 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$ ) on top of a denser sucrose solution ( $\kappa = 0.5 \times 10^{-3} \text{ cm}^2 \text{ s}^{-1}$ ) in a tank 150 mm wide. Note the convective plumes on each side of the thin interface, evidence of strong interfacial transports.

# 海水の密度



密度は温度・塩分・圧力の関数:

$$\rho = \rho(T, S, P)$$

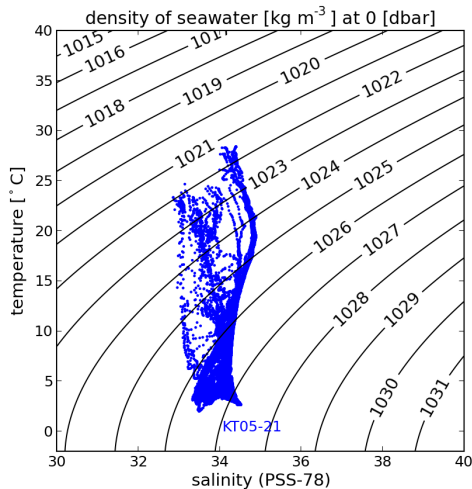
二重拡散対流のように小スケール (~変動量も小さい) の現象では線形近似で充分:

$$\rho \simeq \rho_0 + \alpha(T - T_0) + \beta(S - S_0)$$

非線形性が効く場合  $\rightarrow$  cabbeling



# 海水の密度



密度は温度・塩分・圧力の関数:

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非線形性が効く場合 → cabbeling

実際の海水 (黒潮続流域) の値

# 基礎方程式

基準状態:  $p_0, \rho_0, T_0, S_0$

基準状態の静水圧平衡:  $\nabla p_0 = \rho_0 \mathbf{g}$

状態方程式 (線形):

$$\rho - \rho_0 = \rho_0 \{-\alpha(T - T_0) + \beta(S - S_0)\}$$

運動方程式:

$$\rho \frac{D\mathbf{u}}{Dt} = \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \rho \mathbf{g} + \mu \Delta \mathbf{u}$$

$p$  と  $\rho$  を基準状態  $p_0, \rho_0$  の静水圧平衡  $\nabla p_0 = \rho_0 \mathbf{g}$  のまわりで線形近似する:

$$p = p_0 + p', \quad \rho = \rho_0 + \rho'$$

# 基礎方程式(つづき)

連続の式 (非圧縮):

$$\nabla \cdot \mathbf{u} = 0$$

$T, S$  の保存式:

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa_T \Delta T$$

$$\frac{\partial S}{\partial t} + \mathbf{u} \cdot \nabla S = \kappa_S \Delta S$$

# スケール・アナリシス

- ▶  $T, S$  の成層 (鉛直勾配)
- ▶ 浮力が重要
- ▶ 拡散・粘性が重要

長さ  $d$  (とりあえず)

$T, S$  変動  $d \overline{T_z}, d \overline{S_z}$  ( $\overline{T_z}, \overline{S_z}$ : 平均勾配)

時間  $d^2 \kappa_T$  (運動方程式の時間変化項と粘性項の比較)

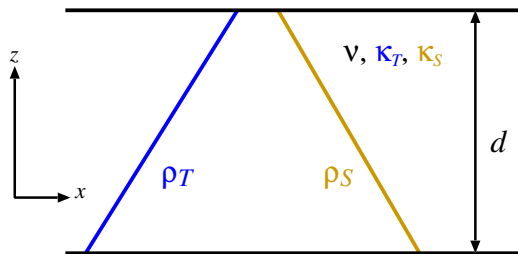
密度変動 (状態方程式)

# ノーマルモード解析

## 問題設定

Baines & Gill (1969)

- ▶ Rayleigh-Bénard 対流の拡張として考える
  - ▶ 水平方向に無限に広い、平行平板間 (間隔  $d$ )
  - ▶ 温度・塩分は境界で固定
  - ▶ 基本場の温度・塩分は勾配一定



# ノーマルモード解析

## 擾乱方程式

線形化した擾乱方程式は

渦度:

$$\frac{1}{Pr} \frac{\partial}{\partial t} \nabla^2 \psi = -R_T \frac{\partial T}{\partial x} + R_S \frac{\partial S}{\partial x} + \nabla^4 \psi$$

熱・塩分:

$$\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial x} = \nabla^2 T \quad \frac{\partial S}{\partial t} + \frac{\partial \psi}{\partial x} = \tau \nabla^2 S$$

無次元パラメータ:

$$Pr = \frac{\nu}{\kappa_T} \qquad \tau = \frac{\kappa_S}{\kappa_T}$$
$$R_T = \frac{\alpha g \Delta T d^3}{\kappa_T \nu} \qquad R_S = \frac{\beta g \Delta S d^3}{\kappa_T \nu}$$

# ノーマルモード解析

固有方程式

変数をノーマルモードに展開:

$$(\psi, T, S) = \text{Re} \left[ (\hat{\psi}, \hat{T}, \hat{S}) \exp(\sigma t + ilx + imy + inz) \right]$$

波数  $l, m, n$  と発達率  $\sigma$  との3次方程式が得られる:

$$\begin{aligned} \sigma^3 + (Pr + \tau + 1)k^2\sigma^2 + [(Pr + \tau Pr + \tau)k^4 + (R_S - R_T)Pr k_H^2/k^2] \sigma \\ + \tau Pr k^6 + (R_S - \tau R_T)Pr k_H^2 = 0 \end{aligned}$$

$$k^2 \equiv (l^2 + m^2 + n^2), \quad k_H^2 \equiv (l^2 + m^2)$$

# ノーマルモード解析

## 固有方程式

さらに擾乱の波数を用いて

$$\sigma' \equiv \frac{\sigma}{k^2}, \quad (R'_T, R'_S) \equiv \frac{k_H^2}{k^6} (R_T, R_S)$$

とスケールしなおすと、

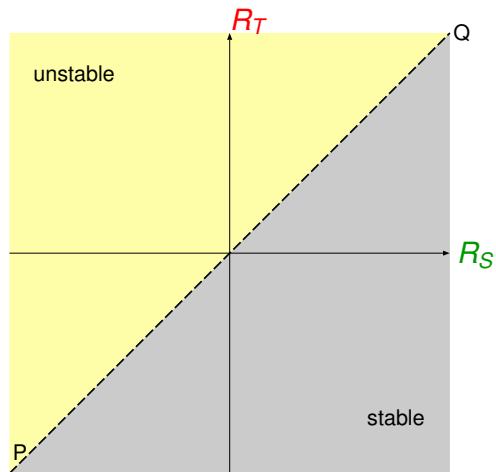
$$\begin{aligned} \sigma'^3 + (Pr + \tau + 1)\sigma'^2 + [Pr(R'_T - R'_S - \tau - 1) + \tau] \sigma' \\ - Pr(\tau R'_T - R'_S - \tau) = 0 \end{aligned}$$



# 発達率方程式の性質

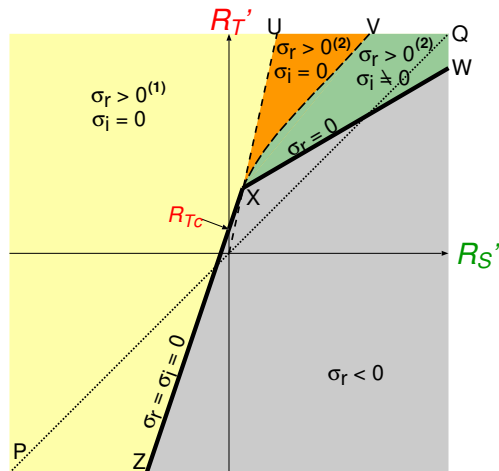
- ▶ 傾いたモード ( $|m| > 0$ ) の発達率は
  - ▶ 鉛直モードの発達率より小さい
  - ▶ 鉛直モードの発達率と同符号
- ▶ 発達率方程式は以下の変換に対して不変 ( $a$ : 定数)
  - ▶  $\sigma \rightarrow a \sigma$
  - ▶  $k \rightarrow a^{1/2} k$
  - ▶  $k_H \rightarrow a^{3/2} k_H$  ( $k_H \equiv \sqrt{l^2 + m^2}$ )

# 線形安定性



(非粘性・非拡散の場合)

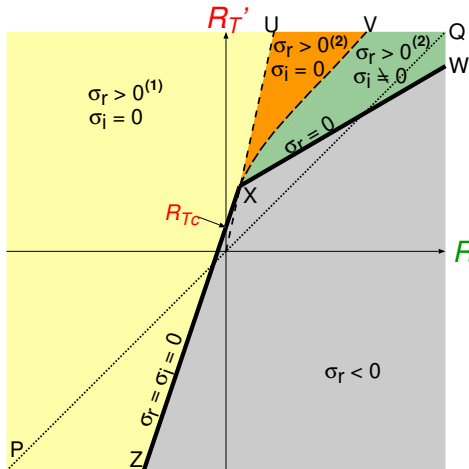
# 線形安定性



解の特性によって分けた領域

- ▶ 安定: 実部が正の解が1つもない
- ▶ 不安定: 正の実解が1つ
- ▶ 不安定: 正の実解が2つ
- ▶ 不安定: 実部が正の複素共役解

# 線形安定性



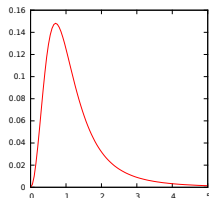
- ▶ ZX:  $\tau R_T' - R_S' = \tau$
- ▶ XW:  $Pr(Pr + 1)R_T' - Pr(Pr + \tau)R_S' = (Pr + 1)(Pr + \tau)(1 + \tau)$

# 線形安定性

前ページの図は擾乱の波数がある値に固定したときのもの。しかし、 $R_T, R_S$  が与えられたときに最も速く増幅するモードを調べたい。

$$\sigma' \equiv \frac{\sigma}{k_H^2 + n^2}, \quad (R'_T, R'_S) \equiv \frac{k_H^2}{(k_H^2 + n^2)^3} (R_T, R_S)$$

だった。 $k_H$  を変えると  $\frac{k_H^2}{(k_H^2 + n^2)^3}$  は



のように変化する。最大値は  $k_H^2 = \frac{n^2}{2}$  のとき  $\frac{4}{27} n^2$  である。

## 線形安定性

また、発達率は  $n = 1$  のときが最も大きくなる (ことが示せる)。

したがって、 $(R_T, R_S)$  面での中立曲線群は、 $(R'_T, R'_S)$  面で  $k_H = \frac{1}{2}$ ,  $n = 1$  としたときの中立曲線群に対応する。こうして

求めた中立曲線群は、

- ▶ XZ:  $R_T = \frac{1}{\tau} \left( R_S - \frac{27}{4} \right)$

- ▶ XW:  $R_T = \frac{Pr+\tau}{Pr+1} R_S + (1+\tau) \left( 1 + \frac{\tau}{Pr} \right) \frac{27}{4}$

$R_S \gg 1$  で

- ▶ XZ:  $R_T = \frac{1}{\tau} R_S + O(1)$

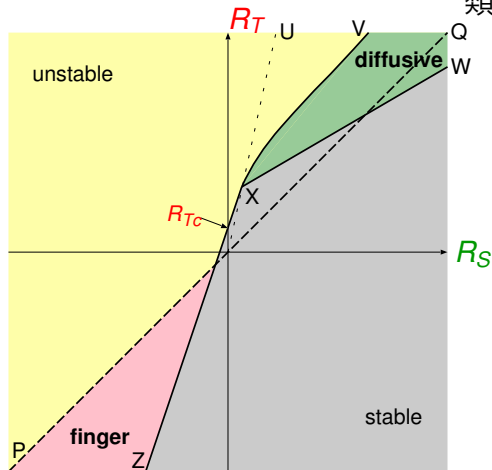
- ▶ XW:  $R_T = \frac{Pr+\tau}{Pr+1} R_S + O(1)$

また、

- ▶ XV:  $R_T = R_S + O(R_S^{2/3})$

も得られる。

# 線形安定性



解の物理的な挙動によって分類しなおす:

- ▶ 単調増幅
  - ▶ 静的不安定 (unstable): 拡散があってもなくても対流
  - ▶ 静的安定 (finger): 拡散のために不安定化
- ▶ 振動的増幅 (diffusive): 静的安定・不安定の両方
- ▶ 減衰 (stable)
  - ▶ 静的不安定: 拡散のために安定化
  - ▶ 静的安定: 拡散があってもなくても安定

# 線形安定性

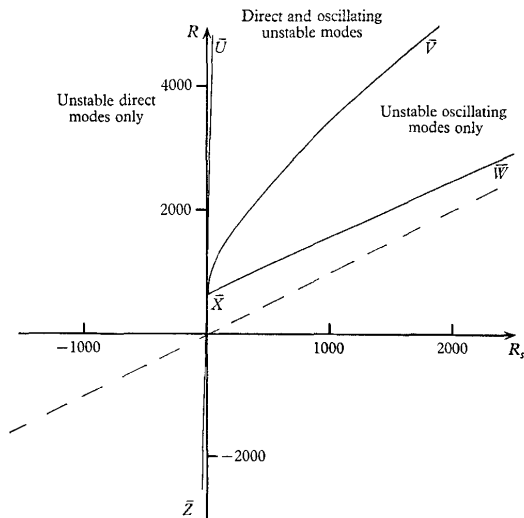


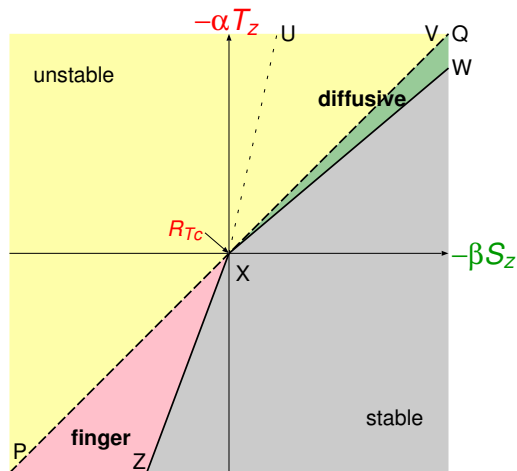
FIGURE 3. Demarcation curves in the  $R, R_s$  plane for regions with different types of unstable modes. The line of neutral buoyancy  $R = R_s$  is shown dashed. \*

\* Baines & Gill (1969)





# 線形安定性

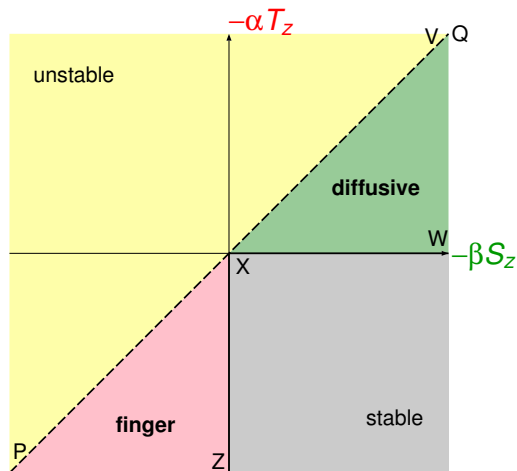


Rayleigh 数  $\rightarrow \infty$

(流体層が  
「非常に深い」場合)

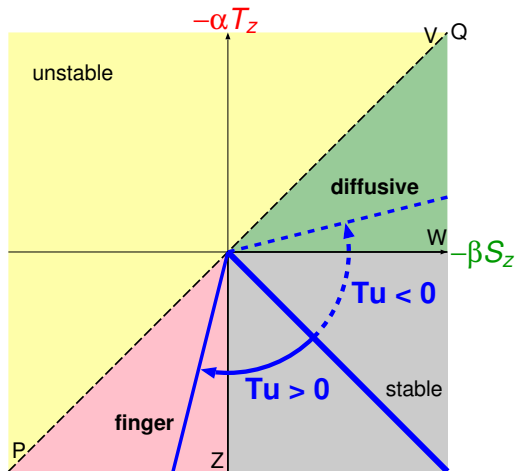
- ▶ XZ:  $\alpha T_z = \frac{1}{\tau} \beta S_z$
- ▶ XW:  $\alpha T_z = \frac{Pr + \tau}{Pr + 1} \beta S_z$
- ▶ XV:  $\alpha T_z = \beta S_z$

# 線形安定性



さらに海洋では、  
 $\tau$  は小さい ( $\sim 10^{-2}$ )  
 $\rightarrow 0$  と近似

# 線形安定性の実用的判定



## Turner 角

$$\tan Tu \equiv \frac{\alpha T_z + \beta S_z}{\alpha T_z - \beta S_z}$$

フィンガー:

$$+45^\circ < Tu < +90^\circ$$

拡散型:

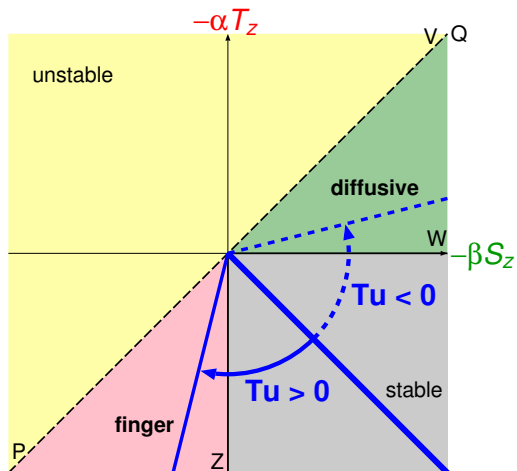
$$-90^\circ < Tu < -45^\circ$$

安定:

$$-45^\circ < Tu < +45^\circ$$

(「最も安定な方向」が  $0^\circ$ )

# 線形安定性の実用的判定



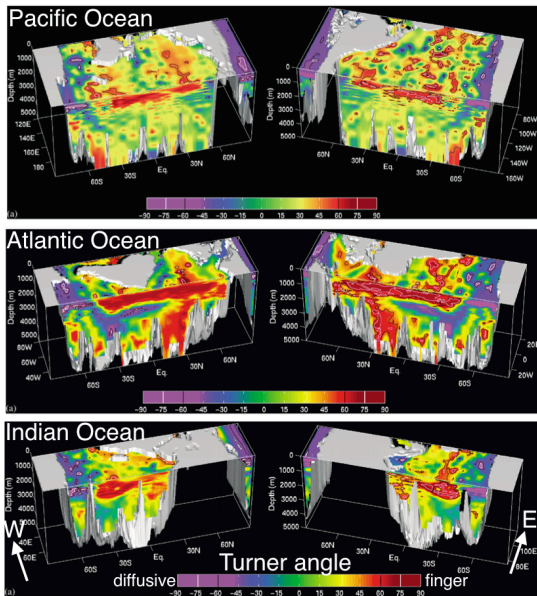
## Turner 角

$$\tan Tu \equiv \frac{\alpha T_z + \beta S_z}{\alpha T_z - \beta S_z}$$

Turner 角の利点:

- ▶  $R_p \equiv \frac{\alpha T_z}{\beta S_z}$  だと  $T, S$  のどちらが安定勾配か分からない
- ▶ 範囲が有限

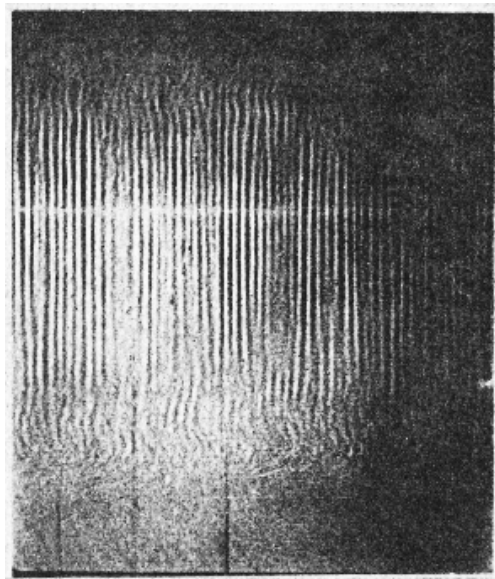
# 線形安定性の実用的判定



実際の海洋での例:  
太平洋・大西洋・インド洋の Turner 角の  
3次元分布 [You,  
2002]

- ▶ フィンガー型: 30%
- ▶ 拡散型: 15%

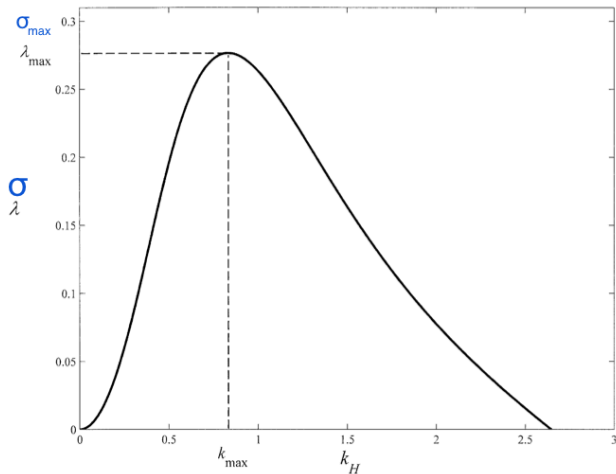
# ソルトフィンガーの構造



[Williams, 1975]

- ▶ 鉛直に非常に細長い (鉛直波数小さい)
- ▶ 明確な水平スケール\*
- ▶ 上下で、屈曲する不安定
  - ▶ 無限に延びることは難しそう

# ソルトフィンガーの発達率



[Radko,2013]



# ソルトフィンガーの構造

平面形状 = 正方形

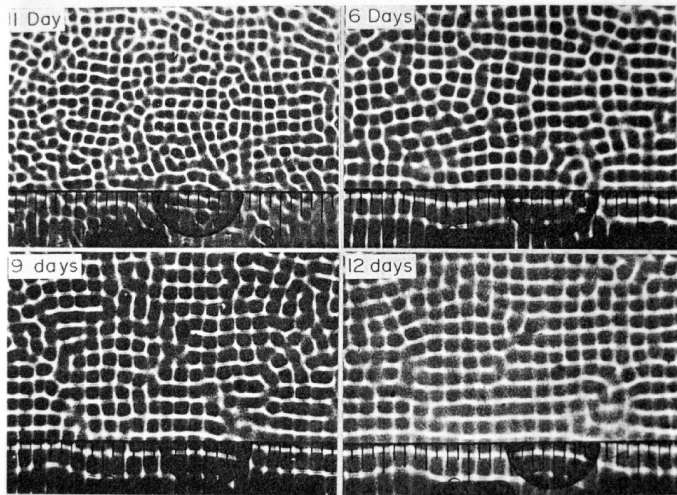


Fig. 6. Top view of sugar fingers. Square packing of fingers is illustrated by the bright lines cast by the fingers in a square pattern. The lines are images of the diagonals of the high index cells. The size and order increases with time.

[Williams, 1975]

# ソルトフィンガーの構造

無限に長いフィンガー

$$\nu \nabla_H^2 w = g(\beta S' - \alpha T')$$

$$w \overline{T_z} = \kappa_T \nabla_H^2 T'$$

$$w \overline{S_z} = \kappa_S \nabla_H^2 S'$$

$$\text{where } T = \overline{T}(z) + T'(x, y)$$

$$\mathbf{u} = (0, 0, w(x, y))$$

解は

$$(w, T', S') = (-w_*, T_*, S_*) \sin \frac{x}{\lambda} \sin \frac{y}{\lambda}$$

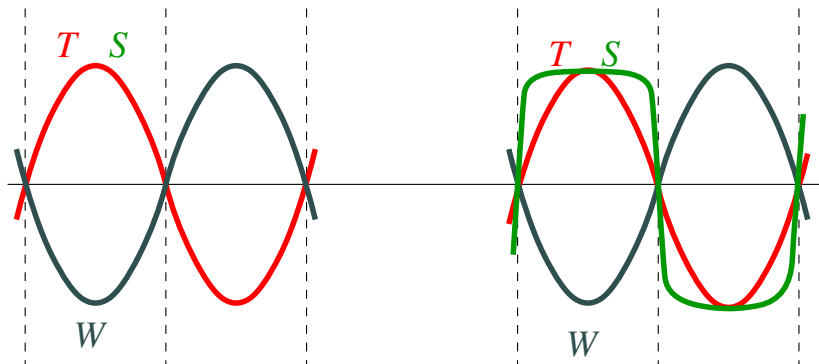
解が存在するための条件は

$$\frac{\alpha g \overline{T_z} \lambda^4}{\kappa \nu} = \frac{4F_R}{F_R + 1} \quad F_R \equiv \frac{\alpha F_T}{\beta F_S} = \frac{\kappa_T \alpha \overline{T_z}}{\kappa_S \beta \overline{S_z}}$$

# ソルトフィンガーの内部構造

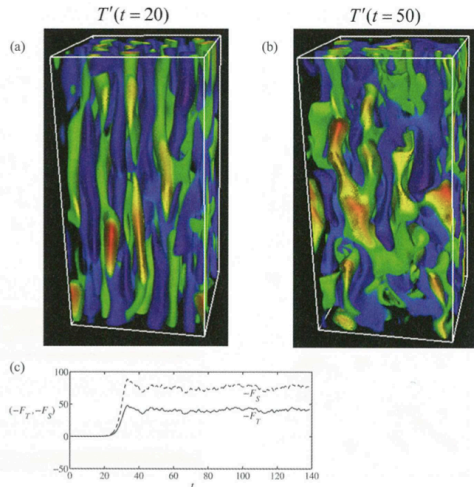
Howard & Veronis (1992)

▶  $\kappa_S \sim 0$  での解



# フィンガーの非線形発展

時間的变化

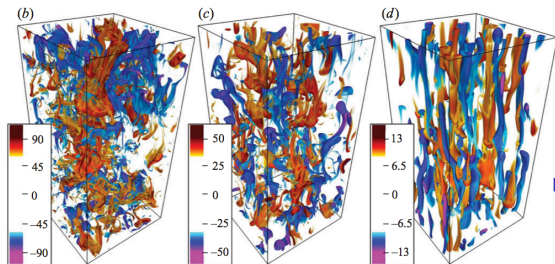


- ▶ 線形発達段階
  - ▶ フィンガーは長く連続
- ▶ 非線形段階
  - ▶ フィンガーは短かく途切れからみあう
  - ▶ 輸送は頭打ち

[Radko & Smith, 2012]

# フィンガーの非線形発展

基本場による発達の違い

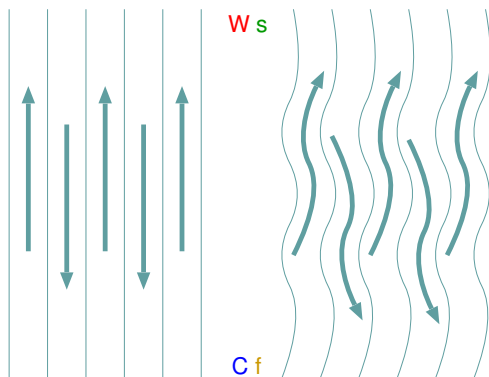


▶  $R_\rho$  が (1 に向かって) 小さくなるにつれて乱流化しやすくなる

FIGURE 1. Snapshots of the salinity field  $S$  in simulations of fingering convection in the heat-salt system ( $Pr=7$ ,  $\tau=0.01$ ). (a) Salinity field at  $R_\rho=1.2$ , plotted on the three planes  $x=0$ ,  $y=0$  and  $z=L_z$ . (b-d) Volume rendering of the salinity field for  $R_\rho=1.2$ ,  $R_\rho=2$  and  $R_\rho=10$  (from left to right). In all cases, the simulation domain contains  $5 \times 5 \times 10$  FGW (see main text). Note how the typical amplitude of the salinity perturbation in a finger is of the order of  $1/R_\rho\tau$ , or, in dimensional terms,  $dS_{oz}/\tau$ .

[Traxler *et al.*, 2011]

# フィンガーの非線形発展



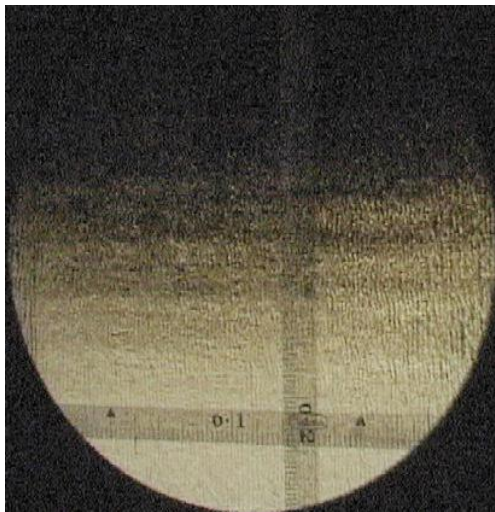
## 線形段階

- ▶ 速度が指数的に増大

## 非線形段階

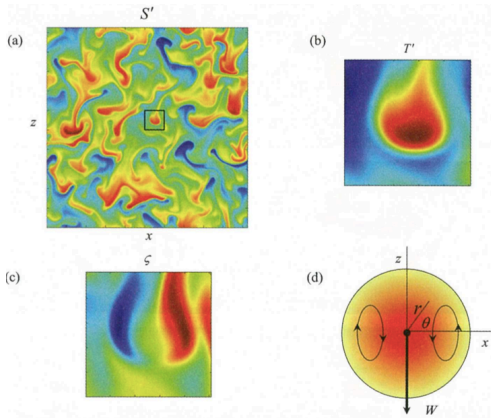
- ▶ せん断が大きくなり、不安定が起る
- ▶ フィンガーは屈曲し、鉛直輸送効率は下がる
- ▶ 準定常状態に達する

# フィンガーの非線形発展



室内実験

# フィンガーの強非線形段階: モドン



[Radko, 2008]

- ▶  $R_\rho$  が 1 に近いときにはフィンガーは途切れ途切れ
- ▶ 浮力を持つ球状の流体塊
- ▶ 球内の運動は渦対
- ▶ これを運動の基本要素と捉える
- ▶ 'double-diffusive modon'



# 拡散型対流の非線形発展

平板間の拡散対流問題の非線形定常解 (Veronis, 1968)

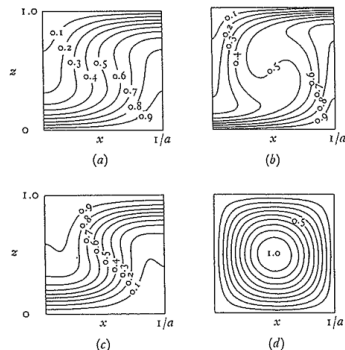


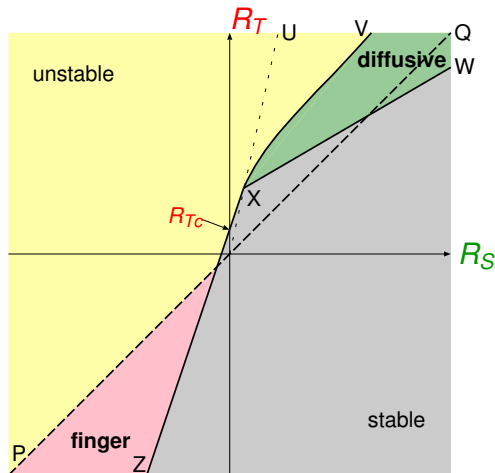
Fig. 8.3. Contours of (a)  $T$ , (b)  $S$ , (c)  $\rho$  and (d)  $\psi$  in a half cell, calculated by Veronis (1968) for the case  $\tau = 10^{-1/2}$ ,  $Pr = 1$ ,  $Rs = 10^3$  and  $Ra = 2500$ . The lines  $x = 0$  and  $x = 1/a$  are lines of symmetry for  $T$ ,  $S$  and  $\rho$  and lines of antisymmetry for  $\psi$ .

設定

- ▶ 二重 Fourier 級数で展開
- ▶ (水平次数)+(鉛直次数) $< K$   $K = 4, 6, 8, 10$
- ▶ 水平波数は線形最大発達モードと同じに

# 拡散型対流の非線形発展

平板間の拡散対流問題の非線形定常解 (Veronis, 1968)



結果

- ▶  $R_{Tcr}$  (線  $XW$ ) の少し上では振動
- ▶  $R_T$  少し増加すると、線  $XV$  より下でも (振動が止んで) 定常対流に遷移
- ▶  $R_S \rightarrow \infty$  ではこの遷移点は  $R_T = R_S$  になる
- ▶  $Pr$  小さくすると、最初から定常モード

# 拡散型対流の非線形発展

平板間の拡散対流問題の非線形定常解 (Veronis, 1968)

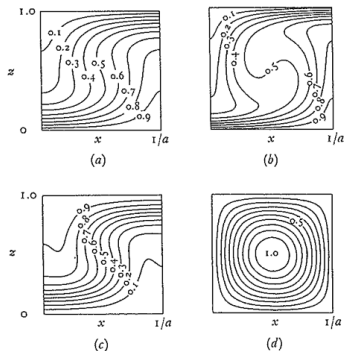


Fig. 8.3. Contours of (a)  $T$ , (b)  $S$ , (c)  $\rho$  and (d)  $\psi$  in a half cell, calculated by Veronis (1968) for the case  $\tau = 10^{-1}$ ,  $Pr = 1$ ,  $R_s = 10^3$  and  $Ra = 2500$ . The lines  $x = 0$  and  $x = 1/a$  are lines of symmetry for  $T$ ,  $S$  and  $\rho$  and lines of antisymmetry for  $\psi$ .

## 結果

- ▶  $R_{Tcr}$  (線 XW) の少し上では振動
- ▶  $R_T$  少し増加すると、線 XV より下でも (振動が止んで) 定常対流に遷移
- ▶  $R_s \rightarrow \infty$  ではこの遷移点は  $R_T = R_s$  になる
- ▶  $Pr$  小さくすると、最初から定常モード

# 拡散型対流の非線形発展

平板間の拡散対流問題の非線形解析 (Veronis, 1965)

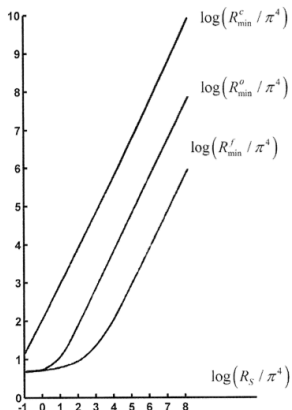


Figure 5.3 The values of three critical thermal Rayleigh numbers  $R_{\min}^c$ ,  $R_{\min}^o$  and  $R_{\min}^f$  (see the text) are plotted as a function of the haline Rayleigh number  $R_S$ . From Veronis (1965).

少し大きいスケールへの影響 (密度輸送)

熱対流による輸送

実験による測定

# 二重拡散対流による鉛直輸送

二重拡散対流によって熱・塩分・密度が鉛直方向に輸送される

- ▶ 一様勾配系
  - ▶ フィンガー対流による輸送
    - ▶ 無限に長いフィンガー
  - ▶ 拡散対流による輸送
    - ▶ 平板間対流
- ▶ 2層系
  - ▶ フィンガー

# 輸送の線形論

## 二重拡散対流による鉛直輸送

- ▶ とくに密度輸送の比  $\gamma \equiv \frac{\alpha F_T}{\beta F_S}$  に興味がある

## 線形理論

- ▶ 振幅は決められない
- ▶ しかし、比は決められる
  - ▶ 輸送:  $F_T = \overline{w'T'} + \kappa_T \overline{T_z}$ ,  $F_S = \overline{w'S'} + \kappa_S \overline{S_z}$
  - ▶ 実際の状況では分子拡散による輸送より対流輸送が卓越:  
 $(\overline{w'T'}, \overline{w'S'}) \gg (\kappa_T \overline{T_z}, \kappa_S \overline{S_z})$
- ▶  $\gamma \sim \frac{\kappa_T \overline{T_z}}{\kappa_S \overline{S_z}}$

# フィンガー輸送の線形論

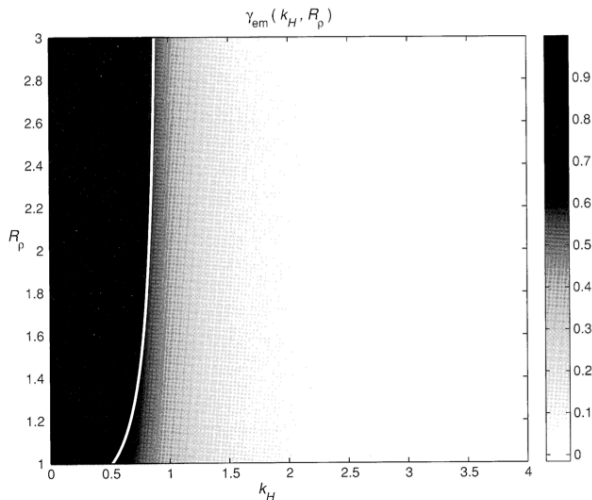
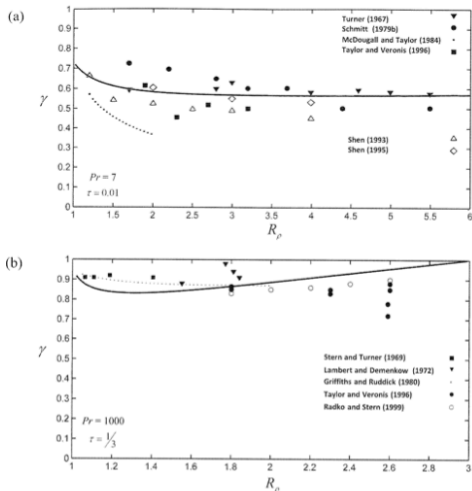


Figure 2.6 Flux ratio of the elevator mode as a function of the wavenumber and the density ratio. Only amplifying modes are shown. The white curve indicates the wavenumber of the fastest growing mode ( $k_{\max}$ ).

[Radko, 2013]



# フィンガー輸送の線形論



実験測定との比較

(a) 熱-塩系

(b) 塩-砂糖系

[Kunze, 2003][Radko, 2013]

Figure 2.7 Salt-finger flux ratio as a function of the density ratio. Symbols are from laboratory and numerical experiments and the solid curves represent the theoretical estimates based on the fastest growing finger model. Calculations are made for (a) the oceanographic (heat-salt) parameters  $(\tau, Pr) = (0.01, 7)$  and (b) salt-sugar parameters  $(\tau, Pr) = (1/3, 10^3)$ . After Kunze (2003).

## 2層系での輸送

この節で考察する状況は

- ▶ 2つのそれぞれは均質な層が重なっている
- ▶ 境界面に二重拡散対流が生じている

しかし、

- ▶ 前節で扱った連続成層と全く異なる
  - ▶ 現実には層に分かれていることが多い
  - ▶ 連続成層から層が形成される場合もある
    - ▶ 層形成のメカニズムは後で考察する
- ▶ 2層系でも局所的に連続成層と考えられる場合もある

# Rayleigh-Bénard 対流による熱輸送

[Priestley, 1954][Howard, 1963]

仮定

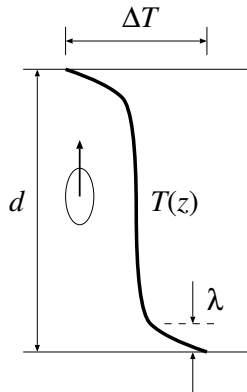
- ▶ 平行平板間の対流
- ▶ 境界層と中心領域に分ける:  
 $d \gg \lambda$
- ▶ 境界層では熱は分子拡散でのみ伝わる

- ▶  $F_T = \kappa \Delta T / \lambda$

- ▶  $Nu \equiv \frac{F_T d}{\kappa \Delta T} = \frac{d}{\lambda}$

- ▶ 一方で、 $Nu = f(Ra)$  なので、  
上を  $Ra$  で表わすと、

- ▶ 
$$Nu = \left( \frac{g \alpha \Delta T}{\kappa \nu} \right)^{-1/3} Ra^{-1/3} d^{-1}$$
$$= \left( \frac{g \alpha \Delta T \lambda^3}{\kappa \nu} \right)^{-1/3} Ra^{-1/3}$$



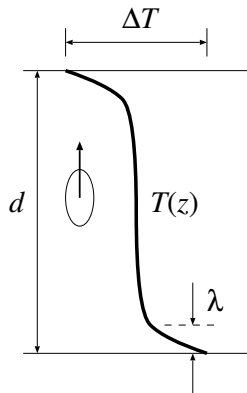
- ▶  $\lambda$  が  $Ra$  に依らないとすると、

$$Nu \sim Ra^{1/3}$$

# Rayleigh-Bénard 対流による熱輸送

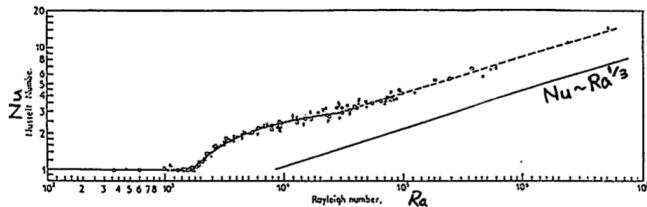
[Malkus, 1963]

- ▶ Priestley, Howard と同じ結果
- ▶ 解釈が異なる
  - ▶  $\frac{g\alpha\Delta T\lambda^3}{\kappa\nu}$  を境界層の Rayleigh 数とする
  - ▶ 境界層は対流が起きるぎりぎりになっていると仮定
    - ▶  $Ra = Ra_{cr} \sim O(10^3)$
    - ▶ ‘中立安定仮説’
  - ▶ つまり、 $Nu \sim (Ra/Ra_{cr})^{1/3}$ 
    - ▶ これは次元量で  $F_T \sim \Delta T^{4/3}$   
(→ 4/3 乗則)



# Rayleigh-Bénard 対流による熱輸送

4/3 乗則 [Chandrasekhar, 1961]



- ▶ 理論も簡単なのでよく使われている
  - ▶ 二重拡散フラックスの表現はほとんどこれ
- ▶ しかし、最近の熱対流実験のデータはこれを支持しない

# Rayleigh-Bénard 対流による熱輸送

[Castaing *et al.*, 1989] (He)

$$Nu \propto Ra^{0.282 \pm 0.006}$$

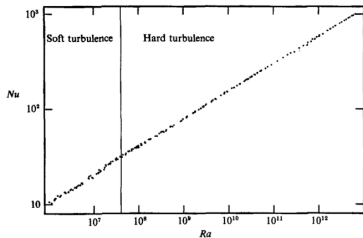


FIGURE 2. The  $Nu$ - $Ra$  correlation. The vertical line at  $Ra = 4 \times 10^7$  indicates the transition from soft to hard turbulence.

→  $Ra^{2/7}$  を提案

$$\frac{1}{3} = 0.33, \quad \frac{2}{7} = 0.286$$

[Kelley, 1990] (水)

$$Nu \propto Ra^{0.284 \pm 0.001}$$

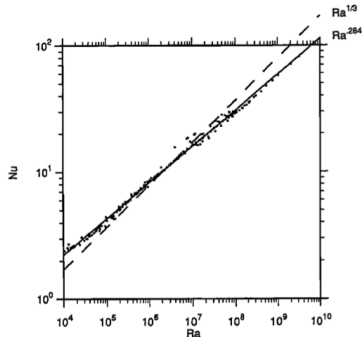
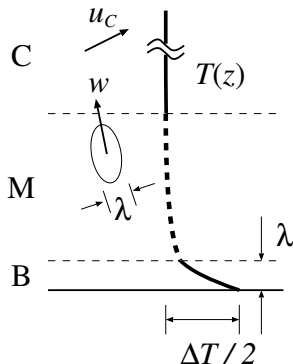


Fig. 5. Heat flux as a function of Rayleigh number, for thermal convection of water in the laboratory. Data sources are as follows: *Chu and Goldstein* [1973]; *Garon and Goldstein* [1973]; *Globe and Dropkin* [1959]; *O'Toole and Silveston* [1961]; *Rossby* [1969]. The solid line is  $Nu = 0.163Ra^{0.284}$  (the best fit of an arbitrary power law) and the dashed line is  $Nu = 0.079Ra^{1/3}$  (the best fit to the traditional 4/3 convection model).

# Rayleigh-Bénard 対流による熱輸送

$Nu \propto Ra^{2/7}$ : Kadanoff の理論 [Castaing et al., 1989]

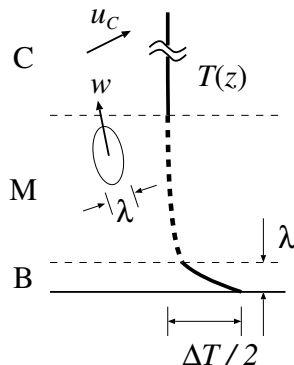
- ▶ 境界層 B は (以前と同様) 熱伝導のみ
  - ▶  $\lambda \sim \frac{\kappa \Delta T}{F_T}$
- ▶ 中心領域 C ではプリュームは自由上昇 (下降)
  - ▶  $u_C \sim \sqrt{g\alpha\Delta T_C d}$
  - ▶  $F_T \sim u_C \Delta T_C \sim \sqrt{g\alpha\Delta T_C^3 d}$
- ▶ B と C の間に混合領域 M がある
  - ▶ 境界層から出たプリュームが加速し、中心領域で浮力・粘性が釣り合う
  - ▶  $w \sim \frac{g\alpha\Delta T_C \lambda^2}{\nu}$



# Rayleigh-Bénard 対流による熱輸送

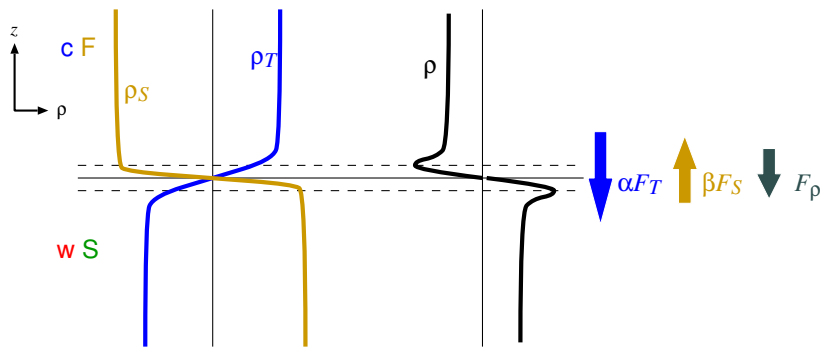
$Nu \propto Ra^{2/7}$ : Kadanoff の理論 [Castaing *et al.*, 1989]

- ▶ これが中心領域の代表速度と等しい
  - ▶  $w \sim u_C$
- ▶ 以上をまとめ、 $T_C$  を消去すると
  - ▶  $Nu \sim Ra^{2/7} Pr^{-1/7}$





## 2層系: 拡散境界面



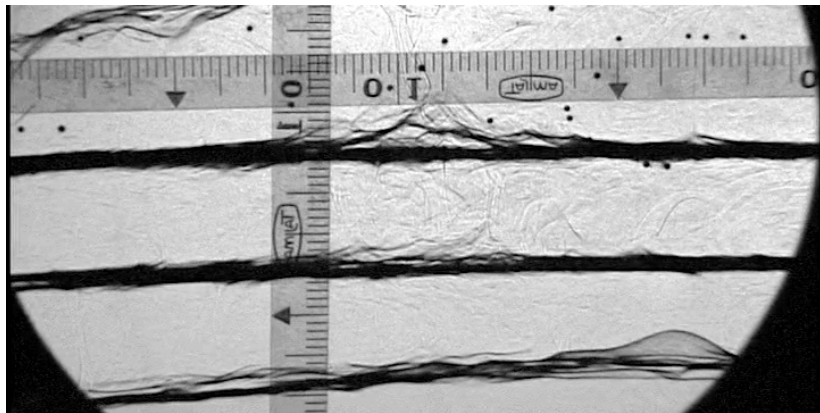
## 2層系: 拡散境界面



Fig. 4. Shadowgraph of a diffusive interface set up by pouring a layer of NaCl solution (the more rapidly diffusing solute) on top of a layer of denser sucrose solution.

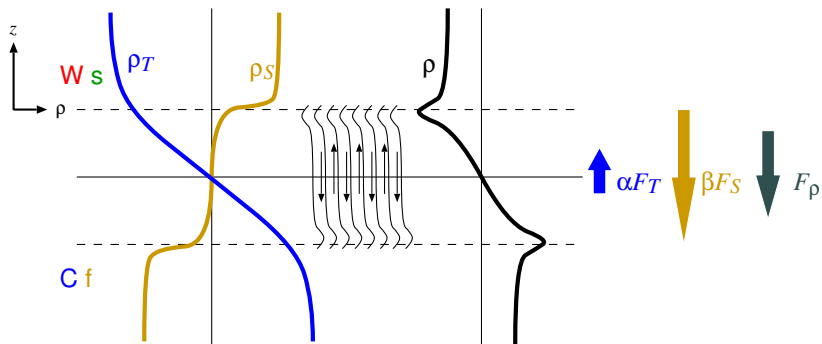
[Turner, 1995]

# 拡散境界のふるまい



movie: diffusive interfaces

## 2層系: フィンガー境界面



## 2層系: フィンガー境界面

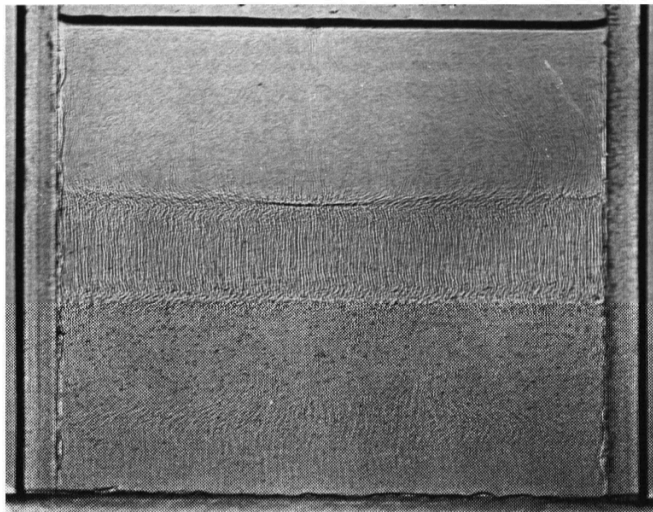
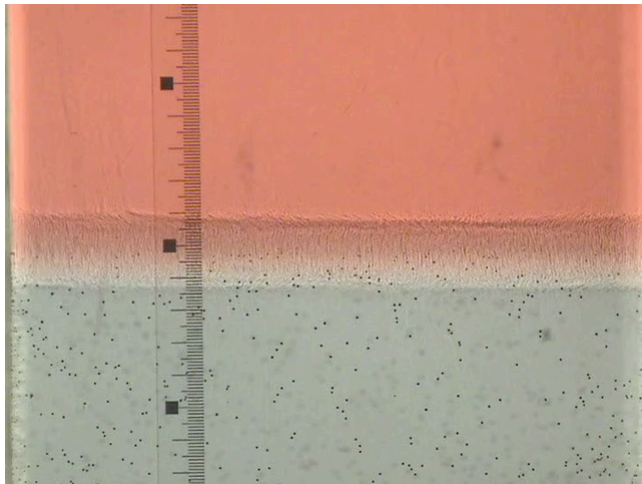


Fig. 6. Shadowgraph of a thickened sugar/salt finger interface formed by placing sucrose solution on top of denser NaCl solution, and leaving for three hours.

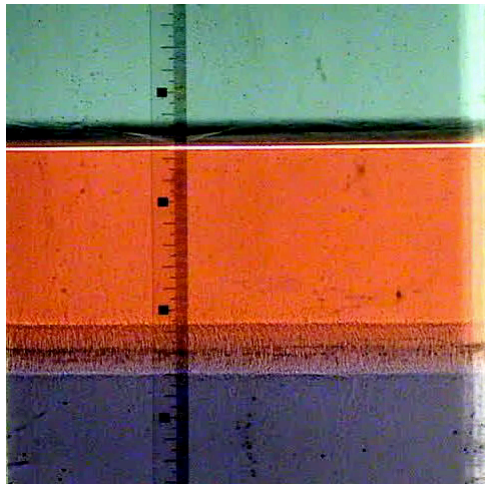
[Turner, 1995]

# フィンガー層の成長



movie: growth of finger layer

# 拡散境界とフィンガー境界



movie: diffusive interface & finger interface

# 実験による測定

測定装置 [Turner, 1965]

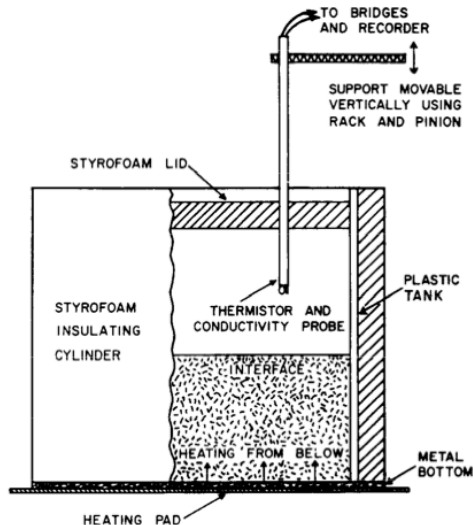


FIG. 3. A schematic diagram of the experimental arrangement.

- ▶ 熱-塩系 (塩-砂糖系ではない)
- ▶ 常に下から加熱



# 拡散境界面での輸送

## 拡散境界面での熱輸送・塩分輸送係数の比

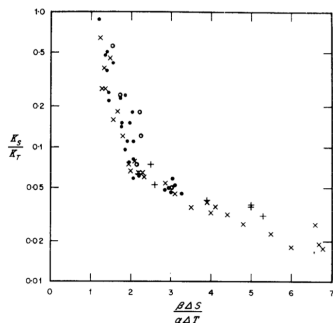


Fig. 5. The ratio of the turbulent transfer coefficients for salt and heat, plotted as a function of the stability parameter  $\beta \Delta S / \alpha \Delta T$ .

[Turner, 1965]

- ▶ 安定性が小さい (密度比  $\sim 1$ ) と  $K_S/K_T \sim 1$ 
  - ▶ 同じ乱流渦によって運ばれる
- ▶ 安定性が大きいと  $K_S/K_T$  は急に減少
  - ▶ 境界面の波打ち \*
  - ▶ 熱は伝導、塩分は碎波
- ▶ 安定性  $\rightarrow \infty$  では  $K_S/K_T \rightarrow \tau$

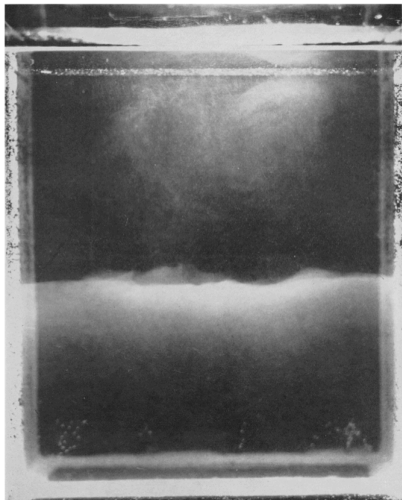
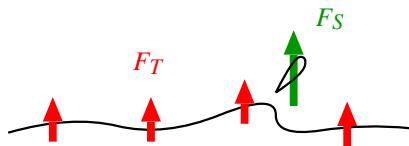


FIG. 2. The two-layer system studied in the experiments reported in this paper.  
The lower, more saline layer has been coloured with fluorescent dye.



[Turner, 1965]

# 拡散境界面での輸送

## 拡散境界面での熱輸送

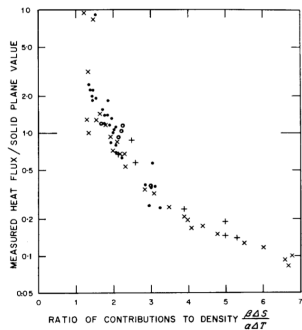


FIG. 4. The measured heat flux across the interface between the two liquid layers, compared with the value calculated for solid planes. This ratio has been plotted against the stability parameter  $\beta\Delta S/\alpha\Delta T$ , the ratio of the contributions of salt and heat to the density difference. In this and the following figures, different symbols have been used for experiments with different heating rates:

○ 1.1 cal/cm<sup>2</sup>/min; ● 1.7 cal/cm<sup>2</sup>/min; + 2.4 cal/cm<sup>2</sup>/min; × 3.3 cal/cm<sup>2</sup>/min.

- ▶ 'solid plate' flux (4/3 則) で規格化
- ▶ 密度比が大きいところでは 1 より小さい
- ▶ 塩分を「持ち上げる」のにエネルギーが使われる

[Turner, 1965]

# 拡散境界面での輸送

拡散境界面での熱・塩分による密度輸送の比

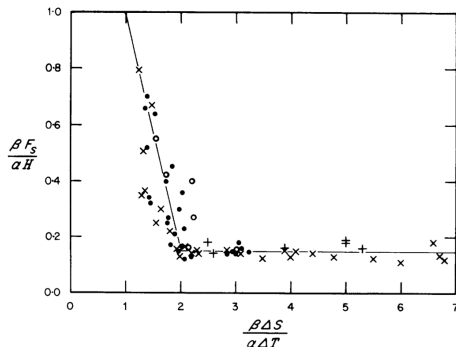


FIG. 7. The ratio of the potential energy changes due to the transfer of salt and heat across the interface, plotted as a function of stability.

- ▶ 密度比  $> 2$  で一定値 0.15
- ▶ 熱輸送によるポテンシャルエネルギーのうち**一定の割合**が塩分の持ち上げに使われる

[Turner, 1965]

# 拡散境界面での輸送

塩-砂糖系 の場合 [Shirtcliffe, 1973]

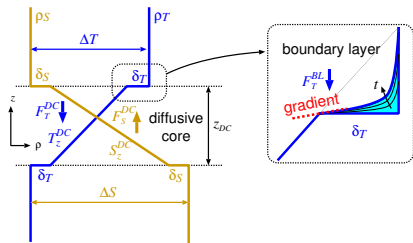
$$\frac{\beta F_S}{\alpha F_T} \sim \left( \frac{\kappa_S}{\kappa_T} \right)^{\frac{1}{2}}$$

- ▶ Linden & Shirtcliffe (1978) による メカニスティックモデル
  - ▶ 定常な境界面
  - ▶  $T, S$  とも同じ厚さ  $z_1$
  - ▶ 分子拡散で厚くなるうとするのと、対流による‘はぎとり’とがバランス



# 拡散境界面での輸送

Linden & Shirtcliffe (1978) による メカニスティック・モデル



$$\blacktriangleright 2\delta_T = \Delta T - T_z^{DC} z_{DC}$$

diffusive core

$$\blacktriangleright F_T^{DC} = \kappa_T T_z^{DC}$$

boundary layer

$$\blacktriangleright T(z) = \delta_T \operatorname{erfc}\left(\frac{z - z_{DC}}{\sqrt{\kappa_T t}}\right)$$

$$\blacktriangleright T_z^{BL} = \frac{2\delta_T}{\sqrt{\pi \kappa_T t}} *$$

$$\blacktriangleright F_T^{BL} = \kappa_T \frac{2\delta_T}{\sqrt{\pi \kappa_T t}}$$

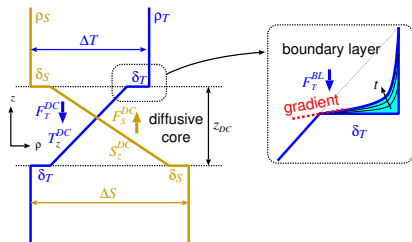
定常状態を仮定

$$\blacktriangleright F_T^{DC} = F_T^{BL}$$

\*  $(\operatorname{erfc} z)' = -2e^{-z^2} / \sqrt{\pi}$

# 拡散境界面での輸送

Linden & Shirtcliffe (1978) による メカニスティック・モデル



さらに以下を仮定

- ▶ 境界層は自らの浮力による対流で取り除かれる
- ▶ プリュームが出たあとは浮力=0
- ▶  $\delta_T = \delta_S$

以上の結果、

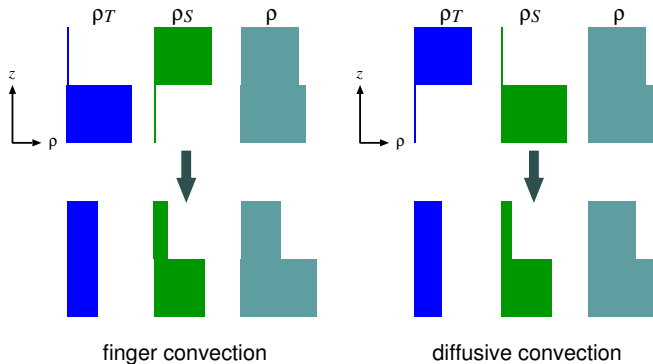
$$\text{▶ } \frac{F_T}{F_S} = \frac{F_T^{BL}}{F_S^{BL}} = \frac{\delta_T}{\delta_S} \sqrt{\frac{\kappa_T}{\kappa_S}} = \sqrt{\frac{\kappa_T}{\kappa_S}} = \tau^{\frac{1}{2}}$$

$$\text{▶ } \left( \frac{F_T}{F_S} = \right) \frac{F_T^{DC}}{F_S^{DC}} = \frac{\kappa_T}{\kappa_S} \frac{T_z^{DC}}{S_z^{DC}} \Rightarrow \frac{T_z^{DC}}{S_z^{DC}} = \tau^{-\frac{1}{2}}$$



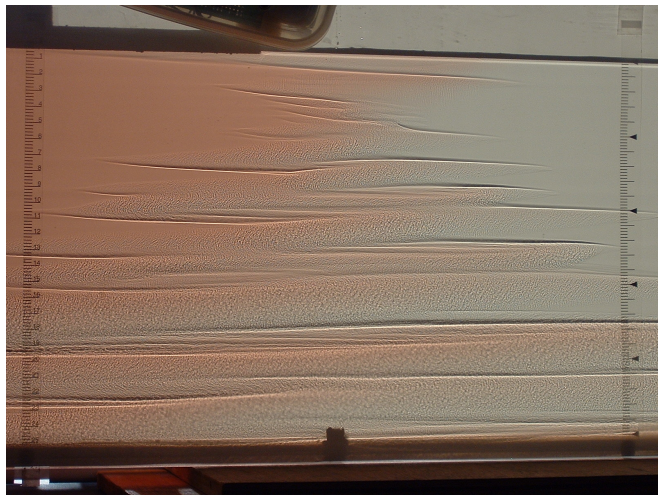
# run-down 問題

- ▶ これまで調べたのは**各瞬間での**輸送特性
- ▶ ここで問題にするのは**最終状態**<sup>†</sup>
  - ▶ 十分時間が経って対流による輸送が**'完了した'**と考える

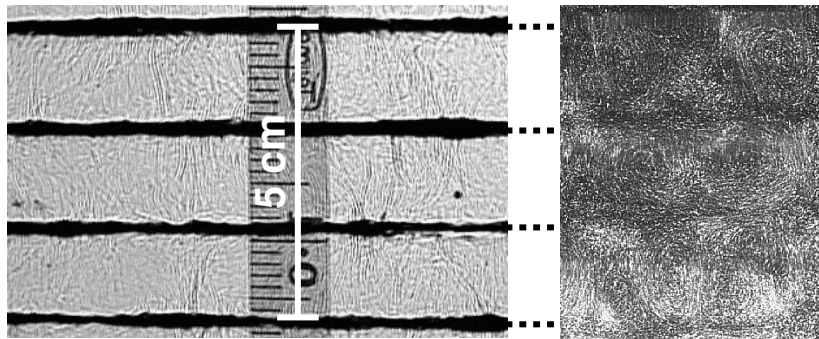


<sup>†</sup>対流がもはや維持できなくなった状態。分子拡散による拡散が止む状態ではない

# 層状構造の形成と発展



# 拡散型対流の多層構造



層の中はセル状の対流運動

# 地質学的な層状構造

## 層状の火成岩

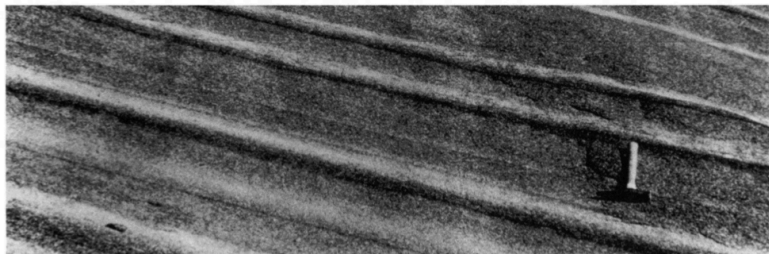
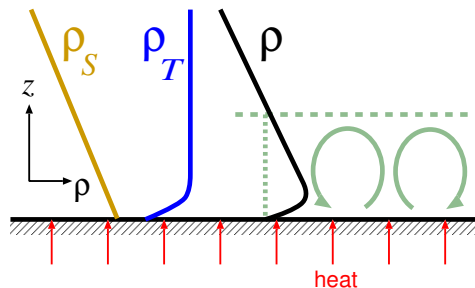


Fig. 14. Layers in the Upper Zone of the Skaergaard igneous intrusion, made visible by contrasting light and dark colours of different minerals (from McBirney and Noyes 1979).

[Turner, 1995]

# 底面加熱



生じる層の厚さ (Chen スケール)

$$H_C = \frac{g\alpha\Delta T}{N^2} = \frac{\alpha\Delta T}{\beta|\overline{S_z}|}$$

# 底面加熱

[Turner, 1995]

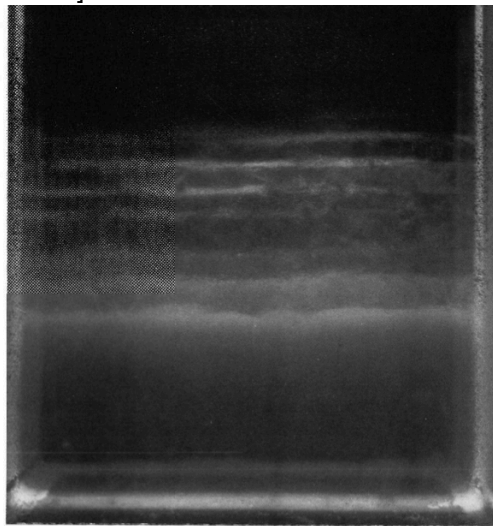
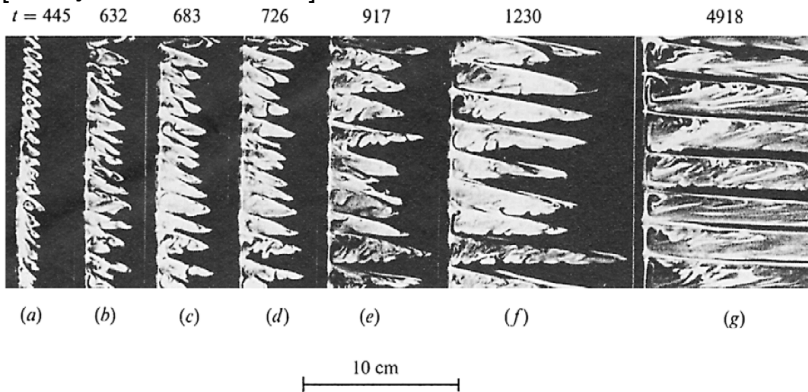


Fig. 3. A series of 'diffusive' interfaces set up by heating a stable salinity gradient from below. The layers are marked by fluorescent dye, and lit from the top.

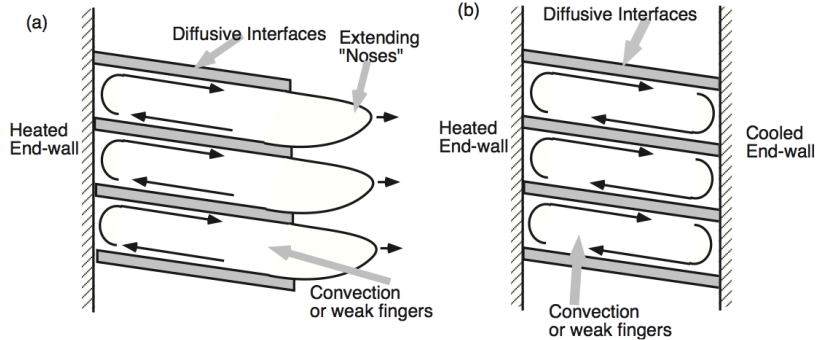
# 壁面加熱

[Tanny&Tsinober, 1988]



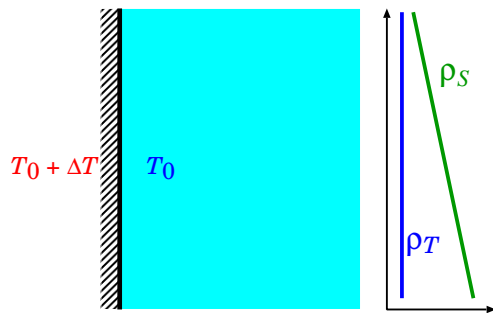
# 壁面加熱

[Thorpe *et al.*, 1969]





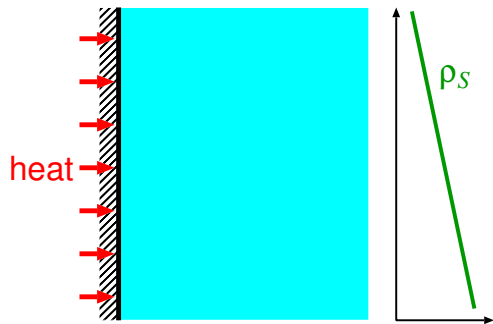
# 壁面加熱：温度一定



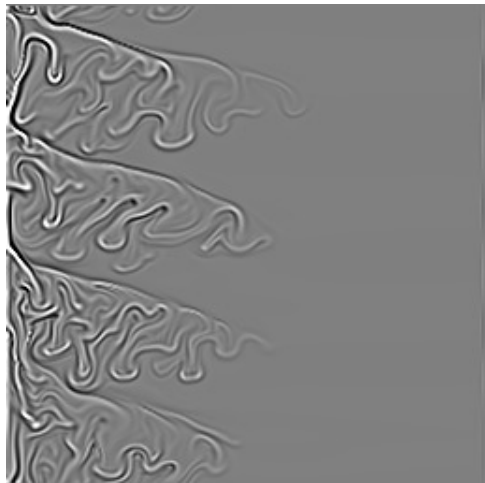
生じる層の厚さ (Chen スケール)

$$H_C = \frac{g\alpha\Delta T}{N^2} = \frac{\alpha\Delta T}{\beta|\overline{S_z}|}$$

# 壁面加熱: フラックス一定

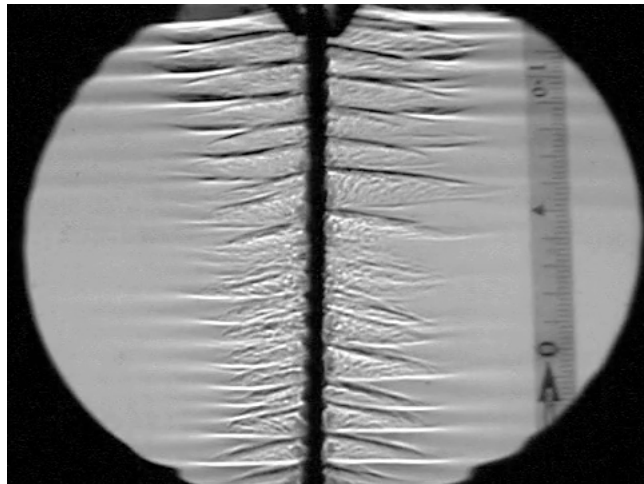


## 壁面加熱: フラックス一定



movie: wall heating simulation

## 壁面加熱: フラックス一定



movie: wall heating experiment

# 斜面の挿入

[Turner, 1995]



Fig. 11. The growth of layers at a sloping boundary in a double-diffusive system with opposing linear gradients (sugar stabilizing, NaCl destabilizing).

## 斜面の挿入

[Prandtl, 1952] 単一成層の場合でも**斜面流**が生じる

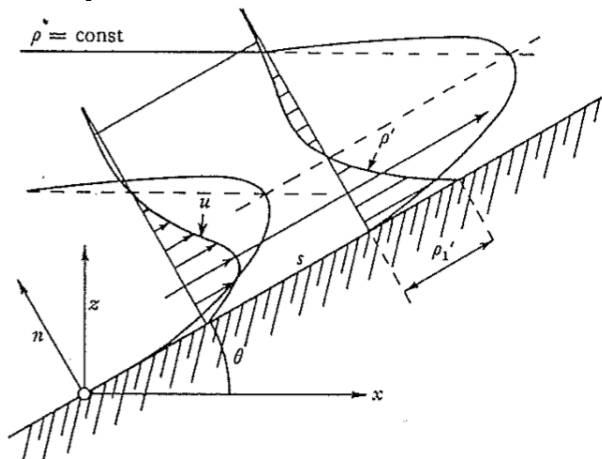
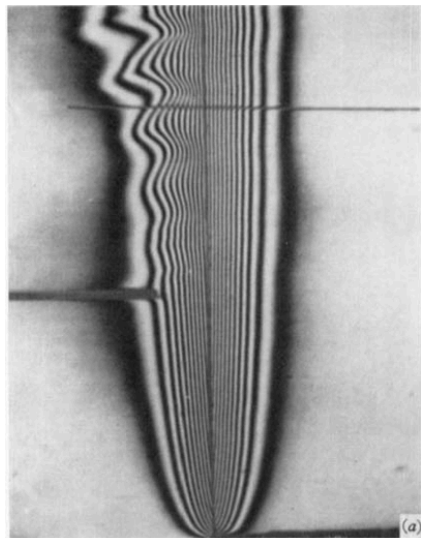
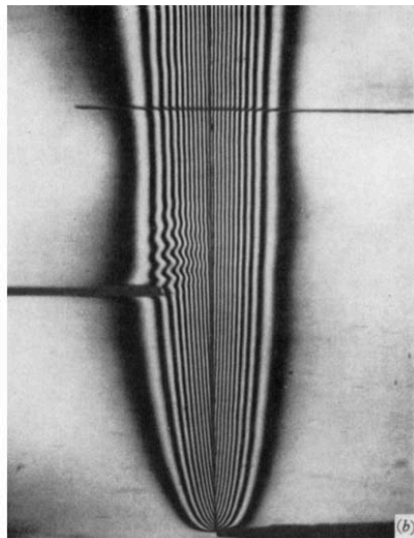


Fig. 7.20. The velocity, density and density anomaly profiles for a buoyancy layer in a linearly stratified environment, with the boundary condition of constant density difference. (From Prandtl (1952), reproduced by permission of Vieweg und Sohn.)

# 鉛直面の自然対流 (成層なし)



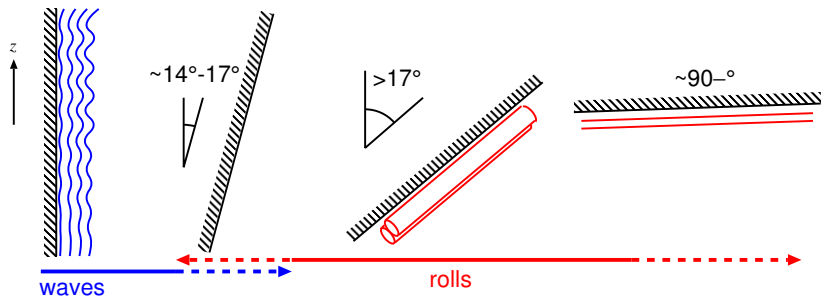
(a) Amplified,  $\alpha = 0.35$ ,  $G = 306$ .



(b) Damped,  $\alpha = 1.56$ ,  $G = 306$ .

[Polymeropoulos&Gebhard, 1967]

# 鉛直面～斜面上の自然対流 (成層なし)





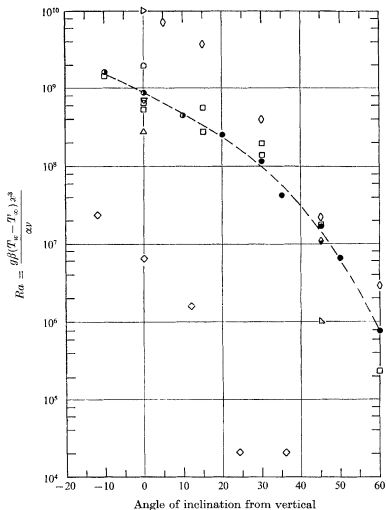


FIGURE 1. Instability Rayleigh numbers. Present investigation (water): ●, vortex instability; ◐, wave instability. □, Lock, Gort & Pond (air). ◇, Tritton (air). △, Kierkus (air). Uniform heat flux: ◇, Vliet (water); ◆, Vliet (air). △, Eckert & Soehngen (air). ▽, Hermann (air). □, Saunders, (air). △, Szewezyk (water).

[Lloyd&Sparrow, 1970]

# 鉛直面～斜面上の自然対流 (成層なし)

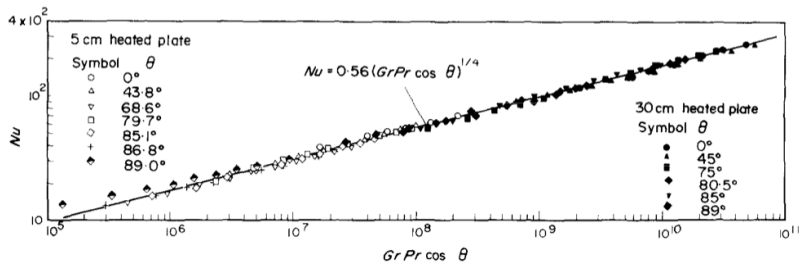


FIG. 6. Relation of  $Nu$  vs.  $GrPr \cos \theta$  for the vertical plate and the inclined plate facing downwards.

[Fujii&Imura, 1972]

# 鉛直壁面からの加熱 (成層なし)

相似解

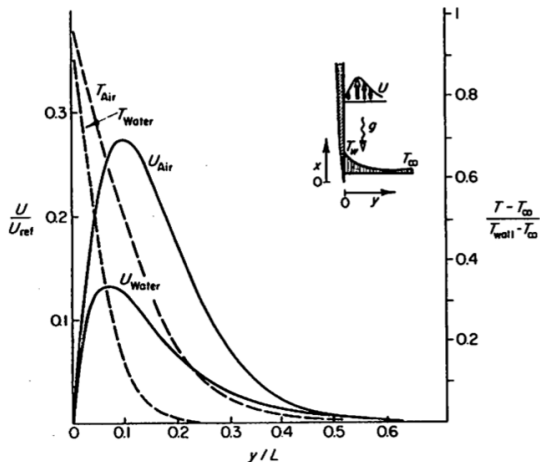


FIG. 42.1. Mean temperature and mean velocity profiles in free convection flow. Air:  $\beta = 0.733$  (from Ostrach, 1953). Water:  $\beta = -6.7$  (from Nachtsheim, 1963).

[Betchov&Criminale, 1967]

# 鉛直壁面からの加熱 (成層なし)

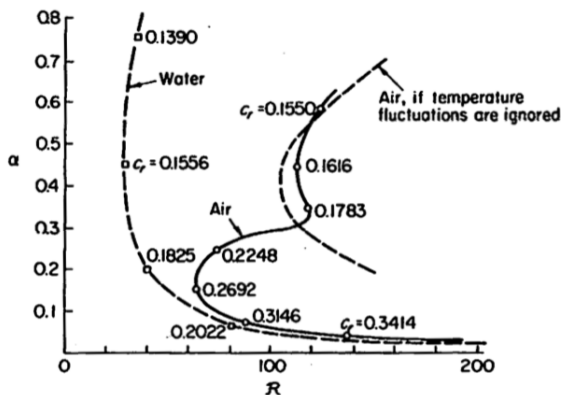
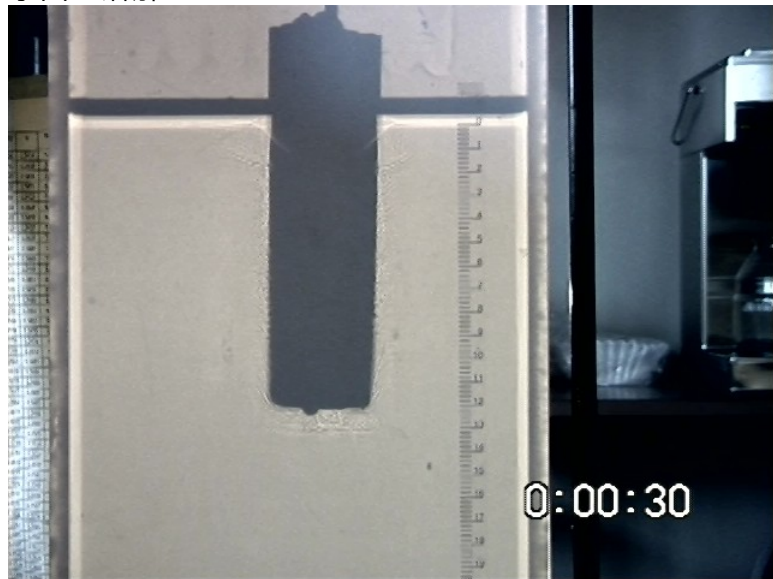


FIG. 42.2. Neutral curves for the free convection flows of Fig. 42.1. (From Nachtseim, 1963.)

[Betchov&Criminale, 1967]

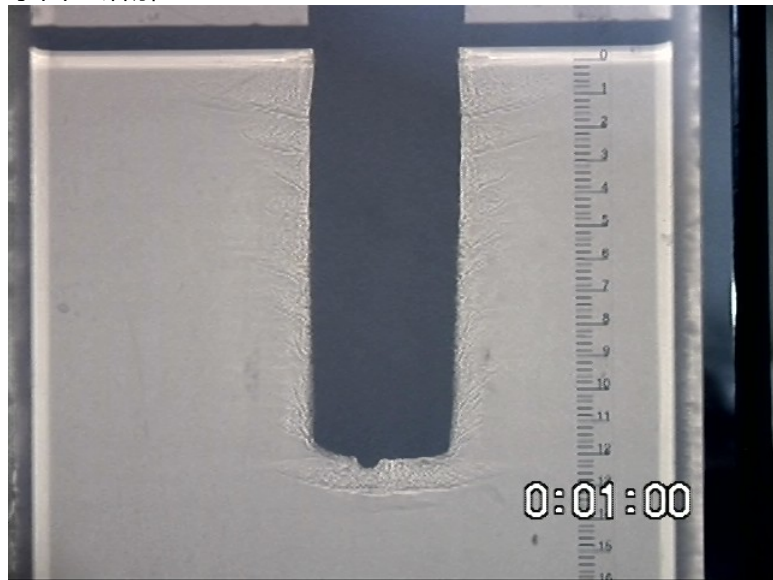
# 壁面での冷却・淡水供給

氷山の溶解



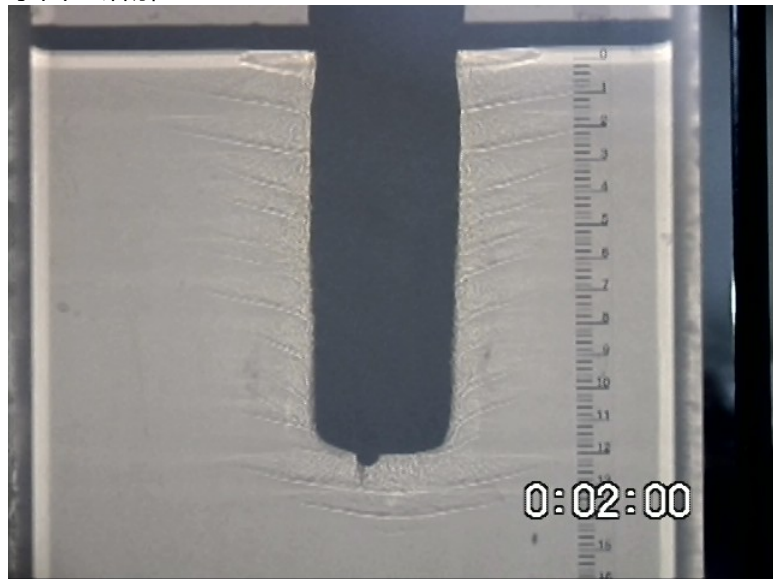
# 壁面での冷却・淡水供給

氷山の溶解



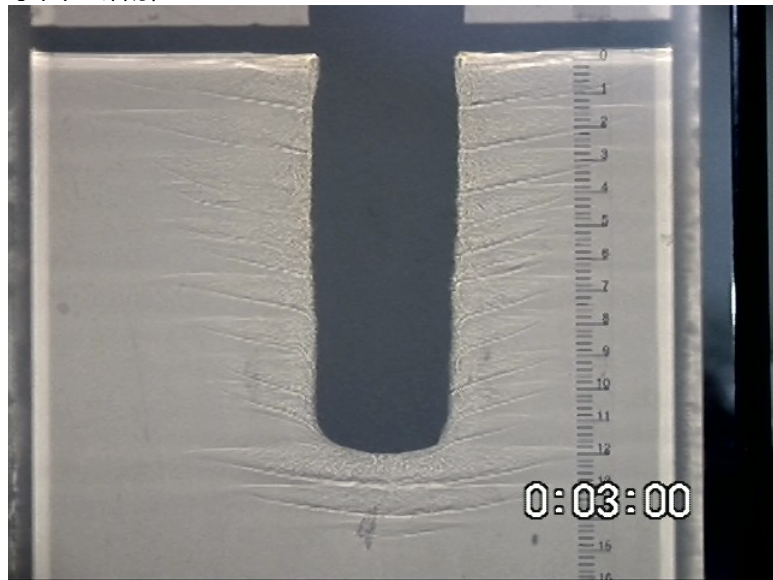
# 壁面での冷却・淡水供給

氷山の溶解



# 壁面での冷却・淡水供給

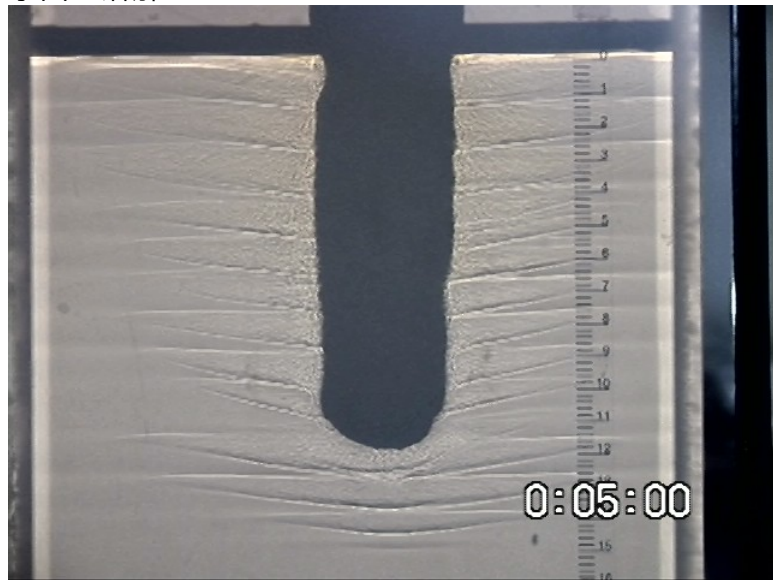
氷山の溶解





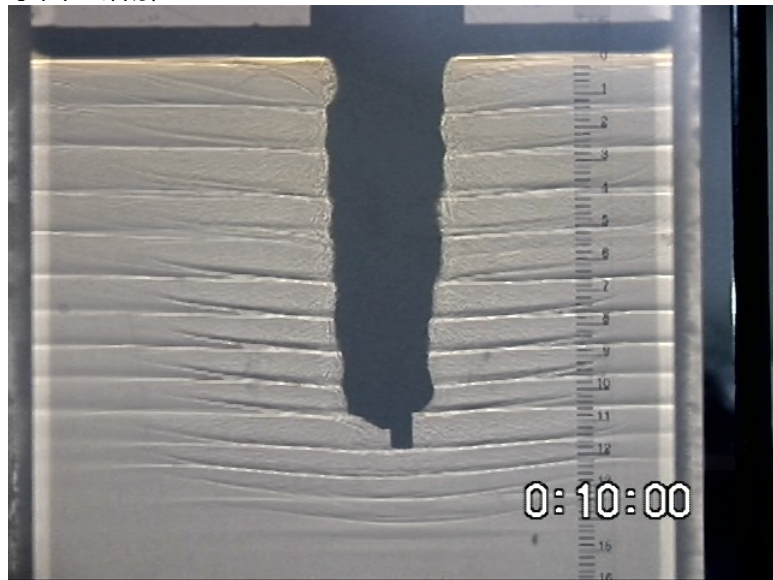
# 壁面での冷却・淡水供給

氷山の溶解



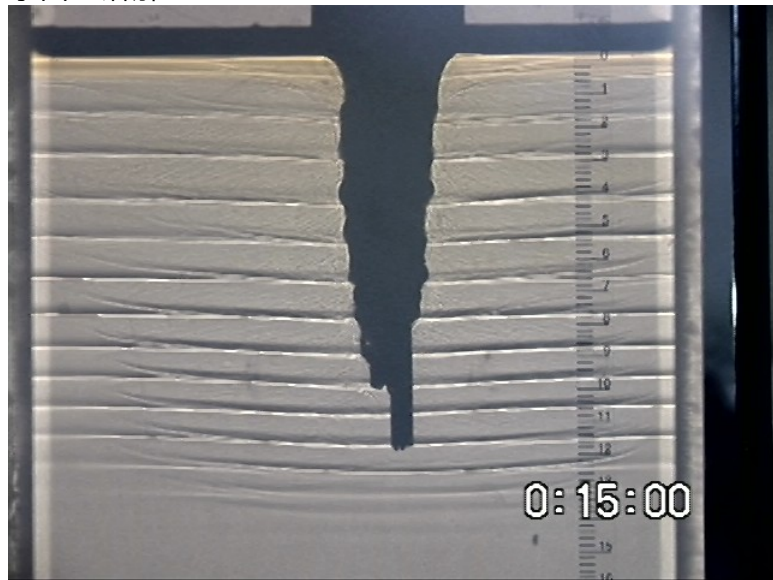
# 壁面での冷却・淡水供給

氷山の溶解



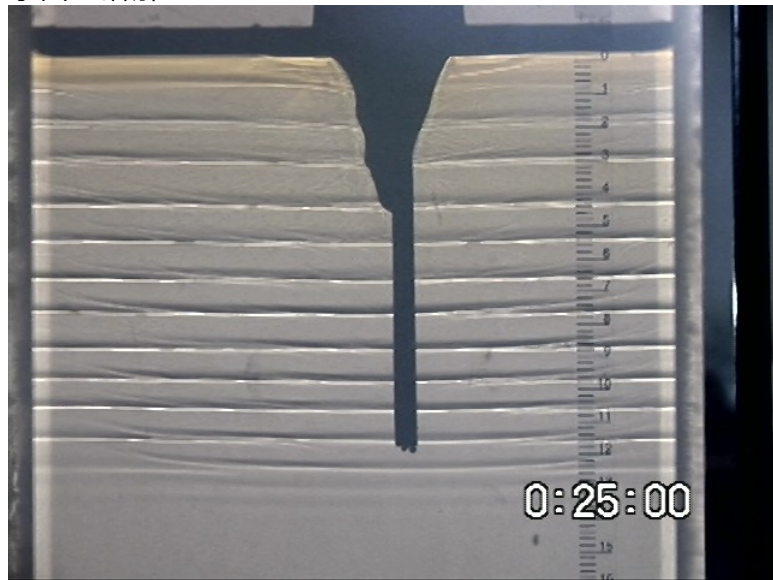
# 壁面での冷却・淡水供給

氷山の溶解



# 壁面での冷却・淡水供給

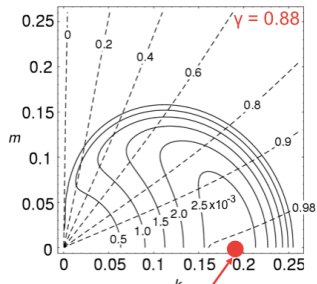
氷山の溶解



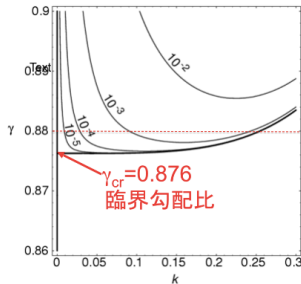
# 線形不安定成層からの層構造の形成

[Noguchi&Niino,2010]

- ▶ 2次元2重周期 (～無限に広い鉛直面) の成層
- ▶ わずかに線形不安定 (密度勾配比 = 0.88 > 0.875(臨界値))



実線: 発達率  
破線: 振動数  
最大発達モード  
 $k = 0.187, m = 0$

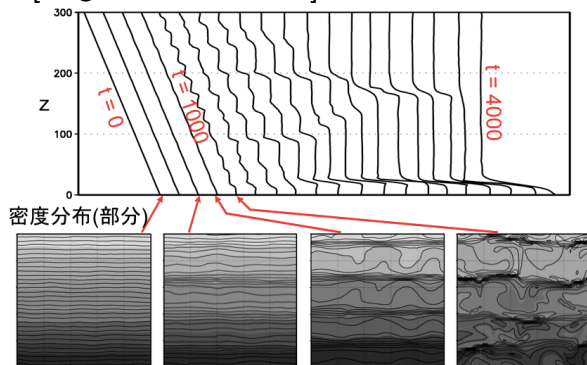


実線: 発達率

- ▶ 線形最大発達モードは柱状 (フィンガー対流と同じ)

# 線形不安定成層からの層構造の形成

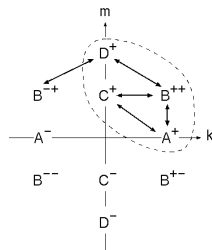
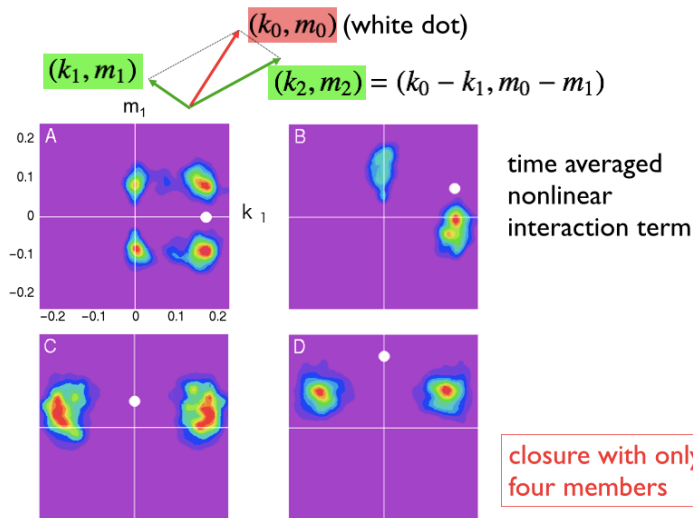
[Noguchi&Niino,2010]



- ▶ 最初は振動する柱状の運動
- ▶ 鉛直方向に密度の粗密が現れる
- ▶ セル状の振動運動
- ▶ セル状の定常対流へ移行

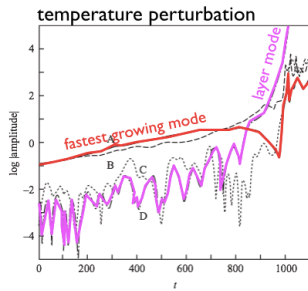
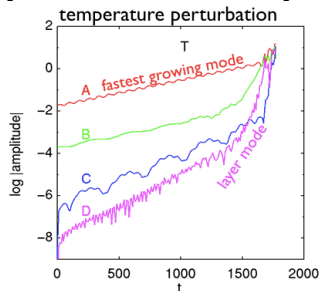
# 線形不安定成層からの層構造の形成

‘発見的’ 打ち切り [Noguchi&Niino,2010]



# 線形不安定成層からの層構造の形成

[Noguchi&Niino,2010]



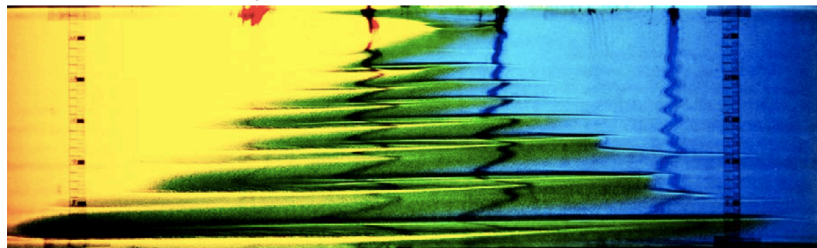
- ▶ 発達モードとの間に共鳴的な関係がある場合、非線形相互作用によって減衰モードも急速に発達する

$$\log |D| \sim \log(|A| \times |B|) = \log |A| + \log |B|$$



# 水平貫入

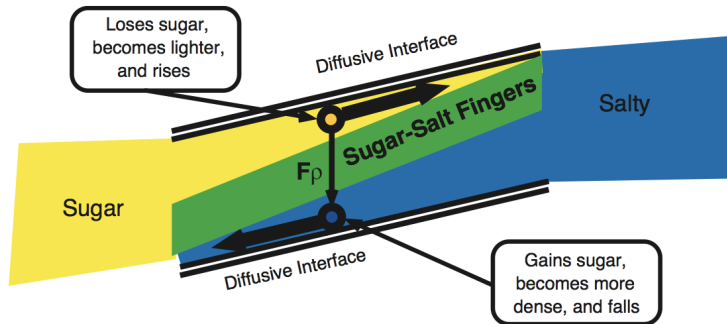
'Ruddick-Turner experiment'



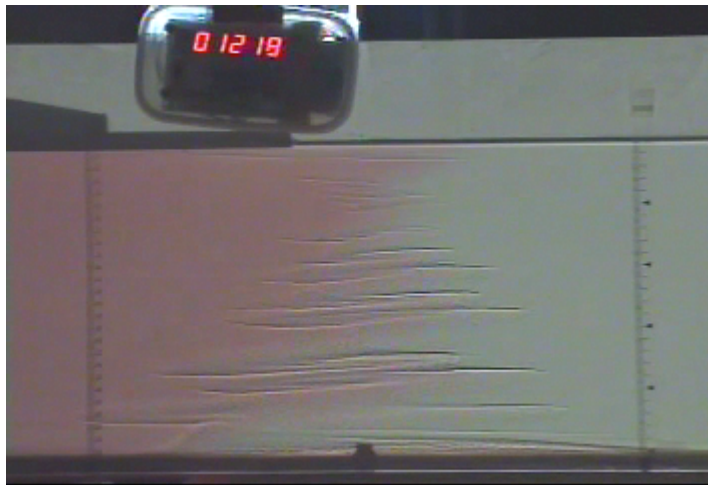
[Ruddick, Philip & Turner, 1999]

# 水平貫入

[Turner, 1978]

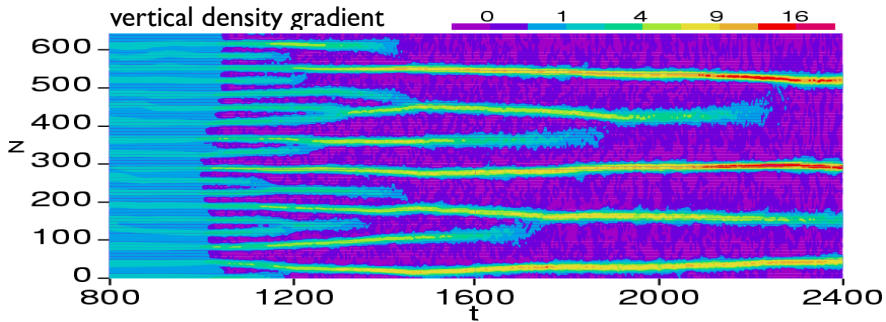


# 水平貫入

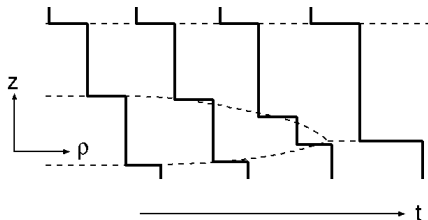


movie: intrusion

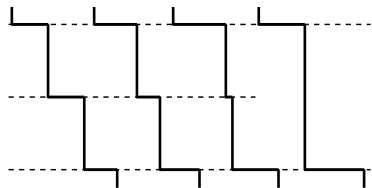
## 2. Growth of layers



layer vanishing mode

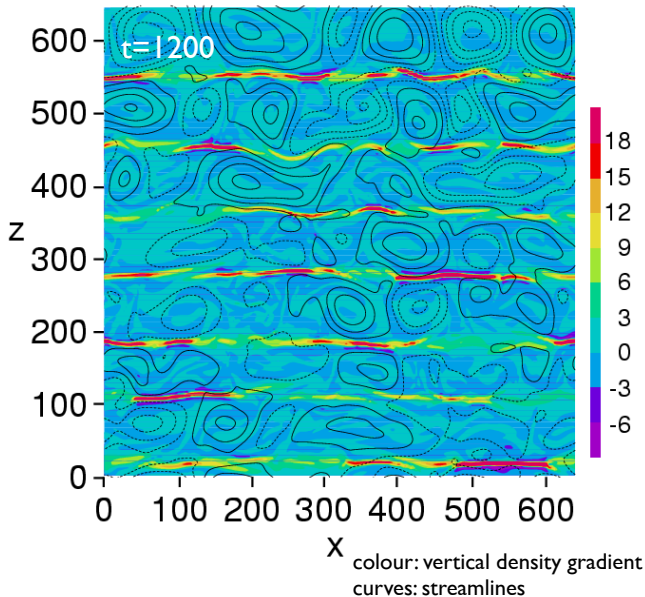


interface vanishing mode



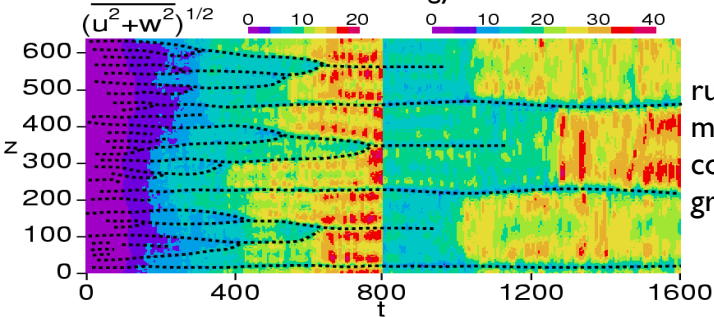
# Structure of multi-layered convection

- cellular convection sandwiched between interfaces
- interaction between neighbouring layers (→ “multi-layered convective system”)



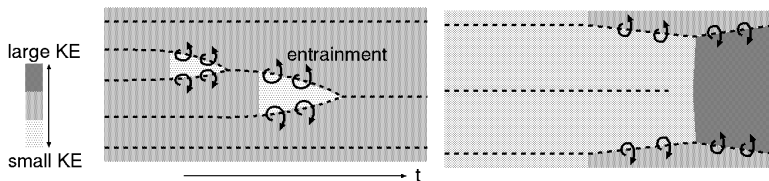
# Layer vanishing and convection

vertical distribution of kinetic energy



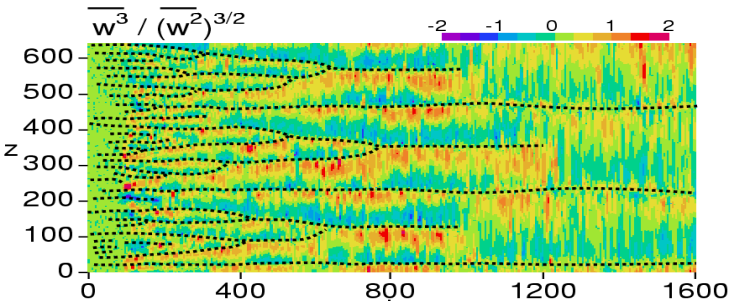
rule I:  
more actively  
convecting layer  
grows

→ can be interpreted as the **competition of entrainment** at interface



# 境界の消滅と層内の運動

鉛直流のskewnessの鉛直分布の時間発展



法則 2:

境界面は層内の運動が上下非対称になると消滅する

→ 境界での密度差の変化も乱流エントレインメントの競合による

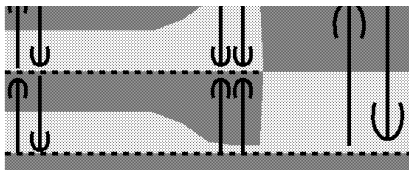
よる

$$\text{skewness}(w) = \frac{\overline{w'^3}}{[\overline{w'^2}]^{3/2}}$$

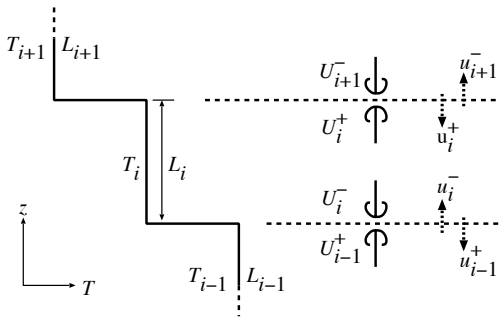
skewness



negative skewness



# Entrainment model



plume velocity at impingement

$$U_i^+ = \sqrt{\Delta T_i^- L_i}$$

entrainment velocity

$$u_i^+ = \frac{U_i^{+3}}{\Delta T_i^+ L_i}$$

$$(\Delta T_i^+ \equiv T_i - T_{i+1})$$

conservation of mass:

$$\frac{dL_i}{dt} = u_i^+ + u_i^- - u_{i-1}^+ - u_{i+1}^-$$



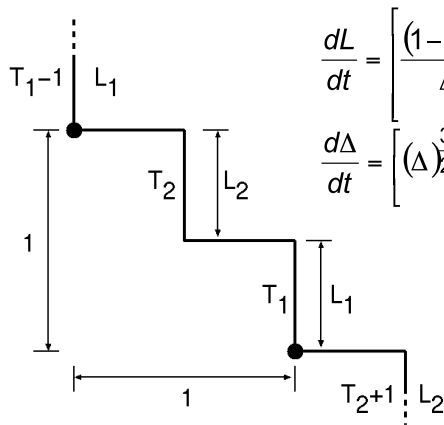
conservation of heat:

$$\frac{dL_i T_i}{dt} = u_i^+ T_{i+1} + u_i^- T_{i-1} - u_{i-1}^+ T_{i-1} - u_{i+1}^- T_{i+1}$$



# 適用例：2層モデル (設定)

2層 (周期的)



$$L = L_1$$
$$\Delta = T_1 - T_2$$

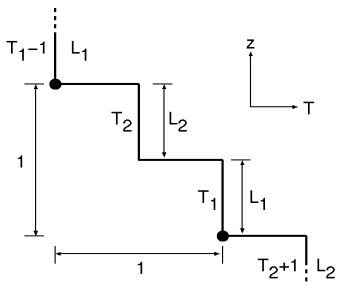
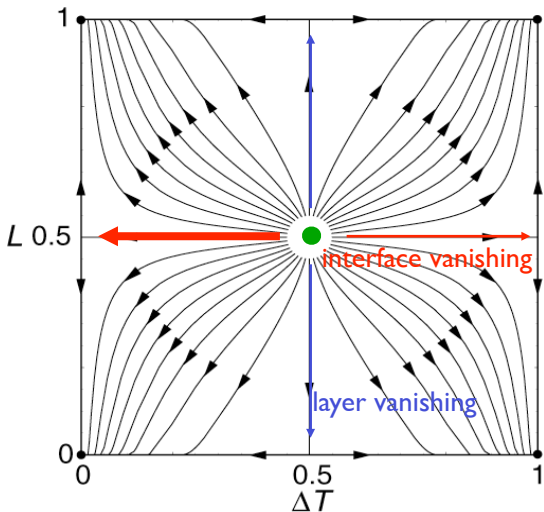
$$\frac{dL}{dt} = \left[ \frac{(1-\Delta)^{\frac{3}{2}}}{\Delta} + \frac{(\Delta)^{\frac{3}{2}}}{1-\Delta} \right] \times \left[ (L)^{\frac{1}{2}} - (1-L)^{\frac{1}{2}} \right]$$

$$\frac{d\Delta}{dt} = \left[ (\Delta)^{\frac{3}{2}} - (1-\Delta)^{\frac{3}{2}} \right] \times \left[ (L)^{\frac{1}{2}} - (1-L)^{\frac{1}{2}} \right]$$

2つの変数だけで状態とその時間変化を表すことができる

# Example I: periodic 2-layer

phase portrait of 2-layer system

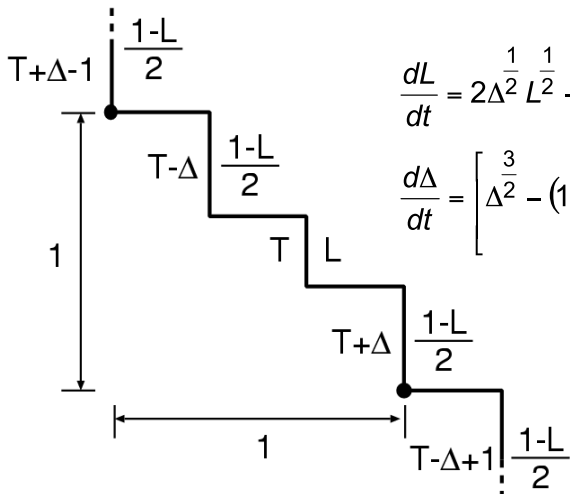


layer merger is inevitable

# 例: 3層モデル (設定)

3層(周期的)

ただし、上下の層は常に同じ厚さとする

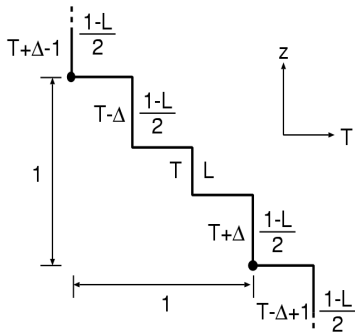
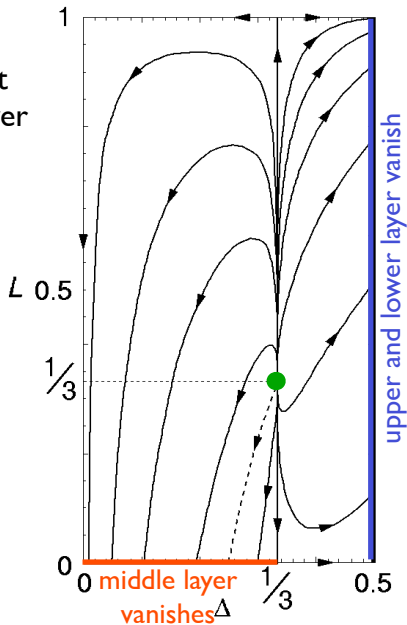


$$\frac{dL}{dt} = 2\Delta^{\frac{1}{2}}L^{\frac{1}{2}} - 2\frac{(1-2\Delta)^{\frac{3}{2}}}{\Delta} \times \left(\frac{1-L}{2}\right)^{\frac{1}{2}}$$

$$\frac{d\Delta}{dt} = \left[ \Delta^{\frac{3}{2}} - (1-2\Delta)^{\frac{3}{2}} \right] \times \left(\frac{1-L}{2}\right)^{-\frac{1}{2}}$$

# Example II: periodic 3-layer

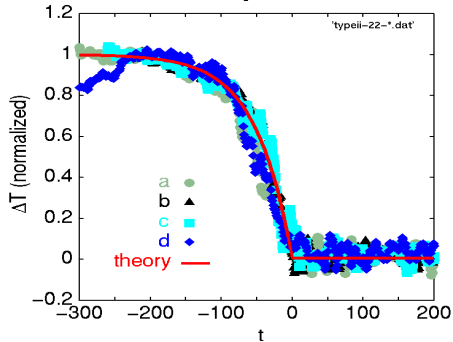
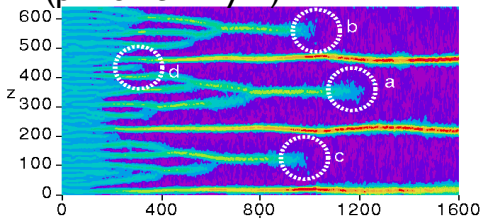
phase portrait of 3-layer system



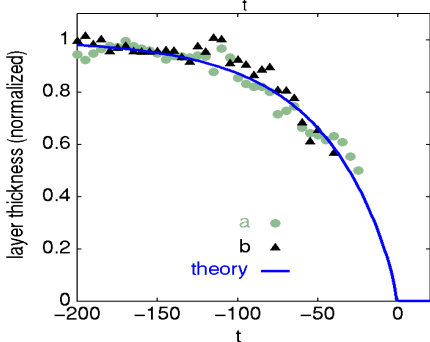
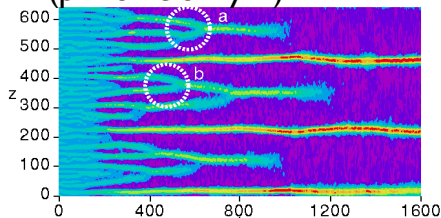
merger is inevitable  
also in 3-layer system

# Entrainment model vs DNS

interface vanishing  
(periodic 2-layer)

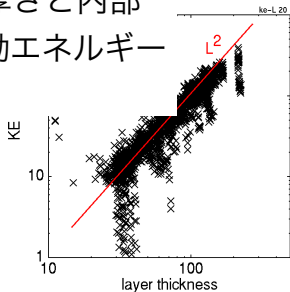


layer vanishing  
(periodic 3-layer)

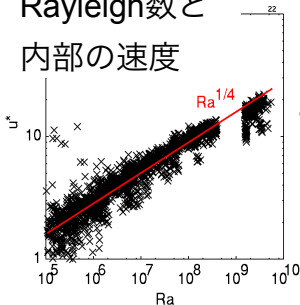


# 多層対流系でのスケール則(I)

層の厚さと内部  
の運動エネルギー



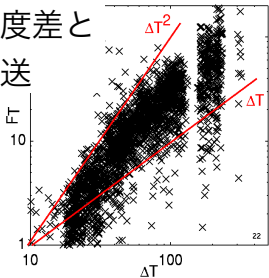
Rayleigh数と  
内部の速度



$$U \approx (\Delta T \times L)^{1/2}$$

と整合的

上下の温度差と  
対流熱輸送



連続成層中の場合でも

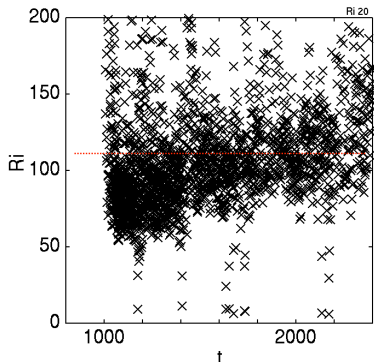
$$F_T \approx (\Delta T)^{4/3}$$

が成り立っているか？

# 多層対流系でのスケール則(2)

境界でのRichardson数の時間変化

層厚の時間変化

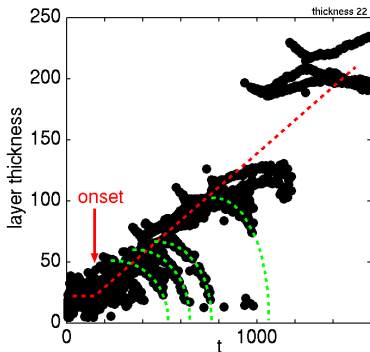


Ri は100程度で

ほぼ一定に保たれている

$$U \approx (\Delta T \kappa L)^{1/2} \Rightarrow Ri = \Delta T \kappa L / U^2 = const.$$

エントレインメント率が一定



(形成からの)時間に比例?

$$u/U \approx Ri = const., \quad U \approx (\Delta T \kappa L)^{1/2} \approx L$$

$$\Rightarrow L/u = const.$$

ある層が消滅するまでの時間は一定

# 星・惑星・地質・工学

星・惑星

地質

工学の応用



# 様々な環境でのフィンガー対流

[Schmitt, 1983]

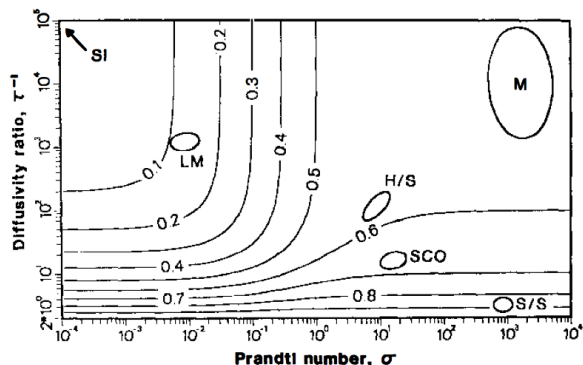
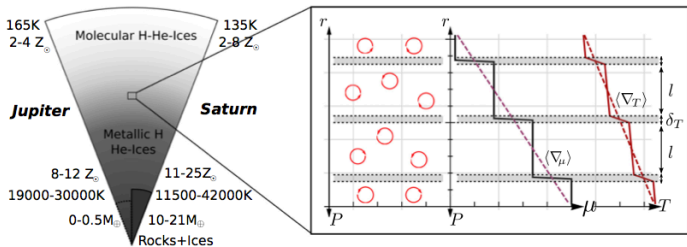


Figure 5 Contours of the flux ratio  $\gamma = \alpha F_T / \beta F_S$  in fingers, calculated for general values of  $\sigma$  and  $\tau$  with  $R_p = 2$  by Schmitt (1983). The regions corresponding to a number of natural double-diffusive systems are denoted by the following abbreviations: LM = liquid metals and metallic semiconductors, SCO = semiconductor oxides, H/S = heat/salt, S/S = salt/sugar, M = magmas. The region for stellar interiors (SI) lies off the diagram at about  $\sigma = 2 \times 10^{-6}$ ,  $\tau = 10^{-7}$ .

# 土星内部の温度構造

## 土星の冷却速度

- ▶ 年齢の割に熱い (赤外放射が強い)
- ▶ 対流による冷却の効率が低いらしい
- ▶ 全層対流ではなく、多層に分かれているのでは?

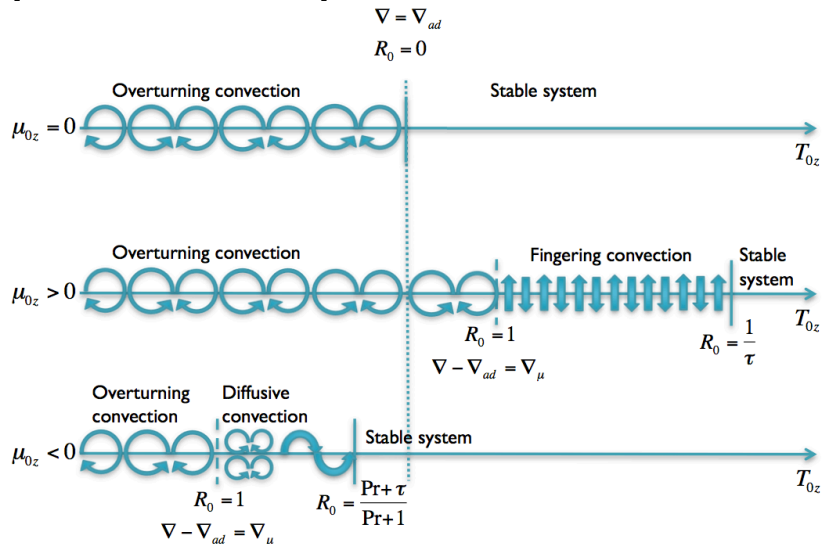


**Fig. 2.** Schematic representation of the interiors of Jupiter and Saturn, according to the present study, and of layered convection, with the resulting compositional and thermal radial profiles. The abundance of metals is constant within the well mixed convective cells of size  $l$ , and undergoes a steep variation within the diffusive interfaces of thermal size  $\delta_T$  (gray regions). Thanks to this steep gradient, these interfaces are stable against convection and energy and matter are transported therein by diffusive processes. Because the size of these layers is very small compared with the size of the planet, the mean thermal and compositional gradients ( $\langle \nabla_T \rangle$  and  $\langle \nabla_\mu \rangle$ ) can be used in good approximation to infer the planet's overall structure.

[Leconte & Chabrier, 2012]

# 天体物理での安定性の分類

[Rosenblum *et al.*, 2011]



# 天体物理での安定性の分類

- ▶ Schwarzschild 条件

$$\nabla - \nabla_{\text{adiabatic}} = \left. \frac{\partial \ln T}{\partial \ln p} - \frac{\partial \ln T}{\partial \ln p} \right|_{\text{adiabatic}} > 0$$

- ▶ 気象学ではふつう  $-\frac{dT}{dz} < \Gamma_d = \frac{g}{c_p}$  と表わす

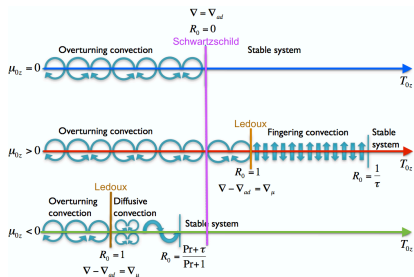
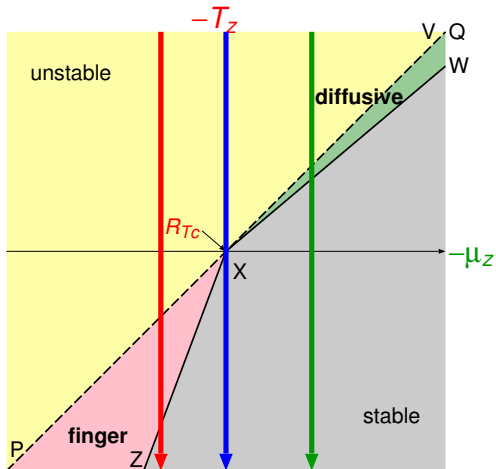
- ▶ Ledoux 条件

$$\nabla - \nabla_{\text{adiabatic}} > \nabla_{\mu} \quad (\mu : \text{平均分子量})$$

$$\iff \left. \frac{\partial \ln T}{\partial \ln p} - \frac{\partial \ln T}{\partial \ln p} \right|_{\text{adiabatic}} > \frac{\partial \ln \mu}{\partial \ln p}$$

- ▶ 組成の勾配がある場合の安定性条件

# 二重拡散安定性の分類との関係



# 星・惑星の特徴

星の内部では光子が熱・運動量の伝導を担うため  $\kappa_T$ ,  $\nu$  が大きい

- ▶  $\tau \sim 10^{-8} - 10^{-6}$
- ▶  $Pr \sim 10^{-7} - 10^{-4}$ 
  - ▶ 海洋の  $\tau \ll 1 < Pr$  と状況が異なる
- ▶ 拡散対流が容易に実現する
  - ▶  $\frac{Pr + \tau}{Pr + 1} \ll 1$

# 二重拡散対流が起こる状況

- ▶ フィンガー対流
  - ▶ 質量の大きい星の表面で H が吹き飛ばされると He が表面付近に溜まる
  - ▶

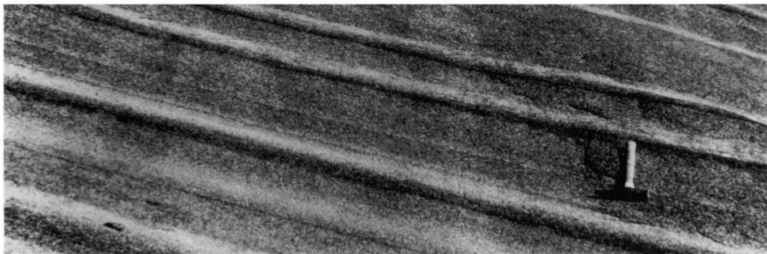


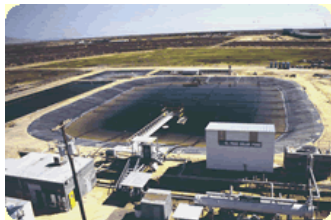
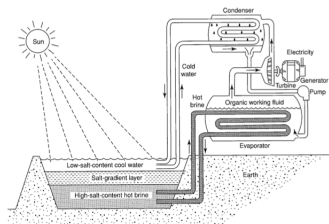
Fig. 14. Layers in the Upper Zone of the Skaergaard igneous intrusion, made visible by contrasting light and dark colours of different minerals (from McBirney and Noyes 1979).



# ソーラーポンド

太陽熱を効率的に集めて貯める池

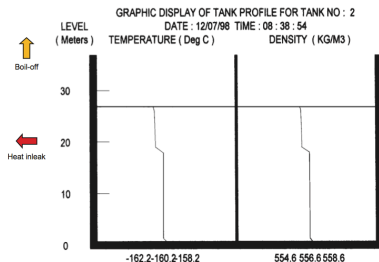
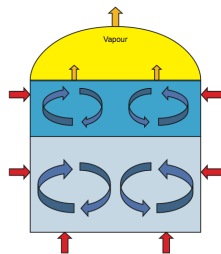
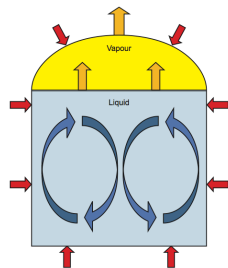
- ▶ 黒い樹脂で内張り (太陽光を吸収)
- ▶ 底層: 濃い塩水 ( $\sim 100^{\circ}\text{C}$ )
- ▶ 表層: 淡水
- ▶ 間に拡散境界面が (自然に) 形成
  - ▶ 対流を抑制
  - ▶ 蒸発による熱のロスを防ぐ
- ▶ 効率 15–25%,  $1\text{ km}^2$  で石油 43000 t/yr に相当 (Tabor, 1980)



# 液化ガスタンクの roll-over

タンク内の液化ガス

- ▶ 壁面からの加熱 → 少しずつ分留 → 組成分離 → 密度成層



- ▶ 層構造が形成 → 鉛直運動を抑制 → 底面近くは過熱
- ▶ 何かの拍子に混合 → 境界面破れる → 過熱流体が急上昇 → 突沸
- ▶ 事故例
  - ▶ 1971年 イタリア La Spezia: LNG 86t 放出
  - ▶ 1993年 イギリス Partington: LNG 350t 放出

# 様々な効果

## 種々の効果

多重拡散

回転

角運動量

セッケン膜

熱拡散現象 (Soret 効果)

気体中の二重拡散対流

状態方程式の非線形性の効果

## 可視化

# 角運動量の拡散

‘角運動量’-‘溶質’系の二重拡散対流

- ▶ 2次元(軸対称)の範囲で完全な対応がある (McIntyre, 1970)
  - ▶ 回転系-成層系のアナロジー (Veronis, 1967)\*の拡張

rotating		$x$	$z$	$u$	$w$	$v$	$T$	$Ek$
stratified		$z$	$x$	$w$	$u$	$T$	$v$	$Ra$

- ▶  $\tau = Pr = 7$ (水)に相当

# 回転系-成層系のアナロジー (Veronis, 1967)

回転系

$$\frac{\partial u}{\partial t} + \epsilon v \cdot \nabla u - 2v = -\frac{\partial p}{\partial x} + E \nabla^2 u$$

$$\frac{\partial v}{\partial t} + \epsilon v \cdot \nabla v + 2u = E \nabla^2 v$$

$$\frac{\partial w}{\partial t} + \epsilon v \cdot \nabla w = -\frac{\partial p}{\partial z} + E \nabla^2 w$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

$$\epsilon \equiv \frac{V}{\Omega L}$$

$$E \equiv \frac{\nu}{\Omega L^2}$$

成層系

$$\frac{\partial w}{\partial t} + \epsilon v \cdot \nabla w - 2T = -\frac{\partial p}{\partial z} + \sigma^{1/2} R \nabla^2 w$$

$$\frac{\partial T}{\partial t} + \epsilon v \cdot \nabla T + 2w = \sigma^{-1/2} R \nabla^2 T$$

$$\frac{\partial u}{\partial t} + \epsilon v \cdot \nabla u = -\frac{\partial p}{\partial x} + \sigma^{1/2} R \nabla^2 u$$

$$\frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} = 0.$$

$$\epsilon \equiv \frac{T}{\Delta T}$$

$$R \equiv \sqrt{\frac{\nu \kappa}{g \alpha \Delta T L^3}} \quad (= (\text{Rayleigh 数})^{-1/2})$$

$$\sigma \equiv \frac{\nu}{\kappa} \quad (= \text{Prandtl 数})$$

# 角運動量の拡散

[Calman, 1977]

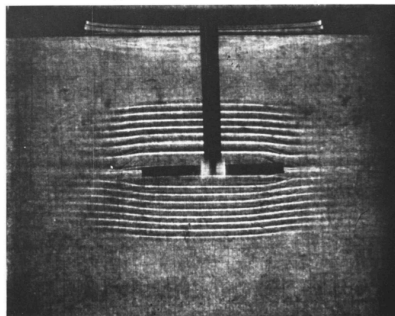


Fig. 3. Density field as seen on a shadowgraph for disc rotation opposite to direction of rotation of the system.

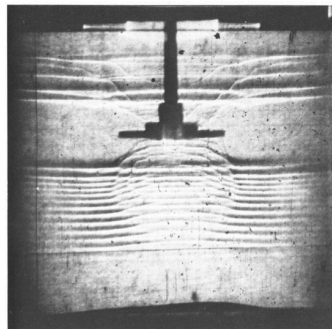


Fig. 6. Density field as seen on a shadowgraph for disc rotation in the direction of rotation of the system.

# セッケン膜

[Martin&Wu,1998]

- ▶ 2次元対流のモデルとしてよく用いられる鉛直セッケン膜で二重拡散対流に**類似**の現象
  - ▶ 温度  $T$ : 加熱による浮力
  - ▶ 厚み  $h$ : 重力による厚み増加 (垂れ下がり)
- ▶ 厚さは重調和方程式に従う (Bruinsma,1995)
- ▶ 運動方程式

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} = \nu \nabla^2 \vec{v} + \frac{1}{\rho_2} \vec{\nabla} \sigma - g \frac{d\rho_2}{\rho_2} \hat{y},$$

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \vec{\nabla} T = \kappa \nabla^2 T,$$

$$\frac{\partial h}{\partial t} + \vec{v} \cdot \vec{\nabla} h = -K \nabla^4 h - a \frac{dT}{dt}.$$

# セッケン膜

[Martin&Wu,1998]

振動型対流

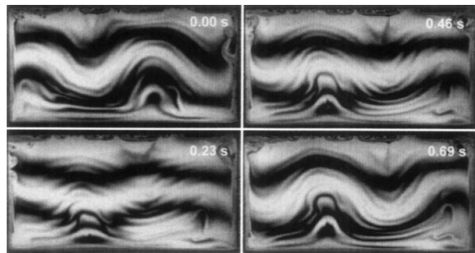


FIG. 1. A sequence of snapshots for the oscillatory instability. The surfactant concentration is  $\phi = 15$  mM, the size of the film is  $1 \times 2$  cm<sup>2</sup>, and  $\Delta T_C = 7$  °C. The interferograms are taken with a sodium lamp at a time interval of 0.23 s. The period of the oscillation is  $\sim 1.4$  s and the wavelength of the instability is  $W/2$ .

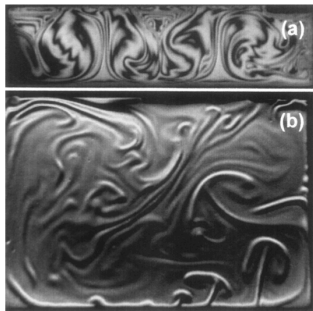


FIG. 2. Flow patterns above the onset. (a) Convective rolls at  $\Delta T \approx 15$  °C with  $d = 0.45$  cm and  $W = 2$  cm, and (b) turbulent flow at  $\Delta T \approx 45$  °C with  $d = 1.5$  cm and  $W = 2$  cm.



# 熱拡散現象

## Soret 効果

- ▶ 温度勾配が濃度勾配を作る

## Dufour 効果

- ▶ 濃度勾配 (化学ポテンシャルの勾配) が温度勾配を作る
- ▶ 溶液中では一般に Soret 効果に比べて小さい

これら 2 つは相反現象

$$J_T = -D_{11} \nabla T - D_{12} \nabla S$$

$$J_S = -D_{21} \nabla S - D_{22} \nabla T$$

Onsager の相反関係:  $D_{12} = D_{21}$

# Soret 効果

[Hurle & Jakeman, 1971]

- ▶ Soret 効果を作る濃度勾配の、対流への影響を調べた
- ▶ メタノール - 水系

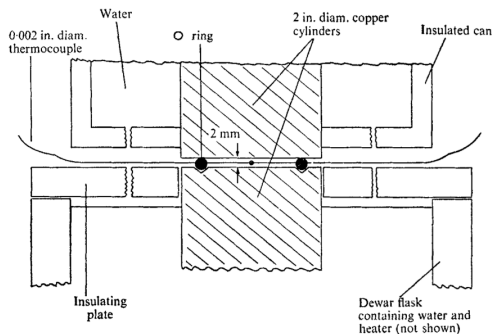
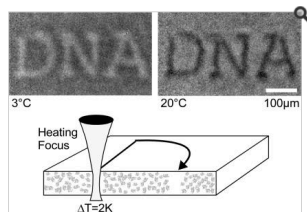


FIGURE 6. Diagram of convection cell (not to scale).

# Soret 効果

Fig. 1.



Thermodiffusion manipulates the DNA concentration by small temperature differences within the bulk solution. A thin water film is heated by 2 K along the letters “DNA” with an infrared laser. For a cooled chamber at 3°C, fluorescently tagged DNA accumulates at the warm letters. However, at room temperature, DNA moves into the cold, showing reduced fluorescence. The chamber is 60 µm thin, containing 50 nM DNA in 1 mM Tris buffer. Every 50th base pair is labeled with TOTO-1 (for details, see [supporting information](#)).

[Duhr&Braun, 2006]

同じ加熱をしても基本温度が異なると粒子が集まったり逃げたりする

注意:

Soret 効果は温度により  
変化する  
(逆転することもある)

# Soret 効果

## メタノール - 水系での Soret 係数

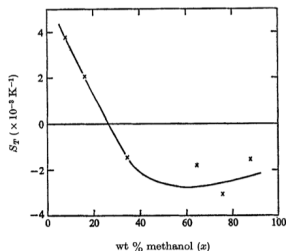


FIGURE 7. Soret coefficient data of Tichacek *et al.* (1956) for water-methanol mixtures at 40 °C.

## 振動モード (overstability)

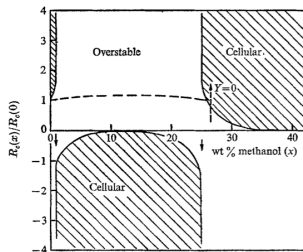


FIGURE 8. Theoretically predicted dependence of critical Rayleigh number on composition for the water-methanol system. The full curves represent the marginal state for stationary convection; the dashed curve that for overstability. The two arrows indicate compositions for which  $\gamma(s/r + 1) = 1$ .

# Soret 効果

## 臨界 Rayleigh 数

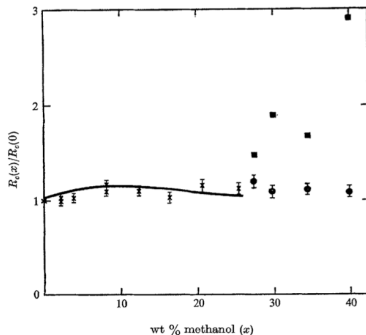
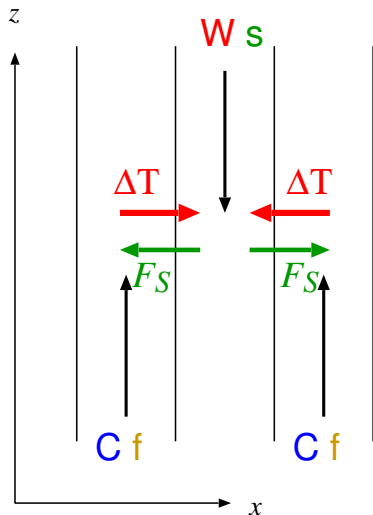


FIGURE 9. Experimentally determined values of the critical Rayleigh number. The curve is the theoretically predicted curve for the onset of overstability. x, Overstable oscillations; ●, stationary convection; ■, finite-amplitude oscillatory modes.

- ▶ 濃度が**薄い**領域で振動対流
- ▶ 理論とよく一致
  - ▶ 二重拡散対流ではない

# Soret 効果

フィンガーの中での塩分に影響する可能性  
[McDougall&Turner, 1982]



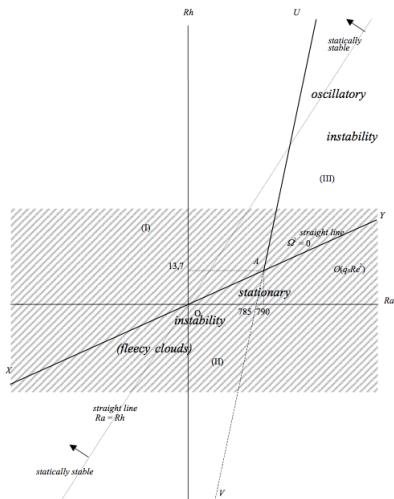
# 気体中の二重拡散対流

観測例はあまりない

- ▶ 大気中では拡散係数の比はそれほど大きくない
  - ▶ 水蒸気:熱 ~ 1:1.2

雲ができる場合 [Bois & Kubicki, 2003]

- ▶ 水滴はずっと‘拡散’が遅い
- ▶ ‘moisture finger’ ができる可能性
  - ▶ ‘fleecy cloud’ の成因



# 気体中の二重拡散対流

(別の実験中に野口がたまたま見つけた)  
ドライアイスの霧のフィンガー状対流



movie: dryice fog finger



# 状態方程式の非線形性の効果

**cabbeling:**  $\partial^2 \rho / \partial \theta^2$

**thermobaric:**  $\partial^2 \rho / \partial \theta \partial p$

# 状態方程式の非線形項

状態方程式:

$$\begin{aligned}\rho = \rho_0 &+ \left(\frac{\partial \rho}{\partial p}\right)_0 (p - p_0) + \left(\frac{\partial \rho}{\partial T}\right)_0 (T - T_0) + \left(\frac{\partial \rho}{\partial S}\right)_0 (S - S_0) \\ &+ \frac{1}{2} \left(\frac{\partial^2 \rho}{\partial T^2}\right)_0 (T - T_0)^2 \\ &+ \left(\frac{\partial^2 \rho}{\partial p \partial T}\right)_0 (p - p_0)(T - T_0) + \dots\end{aligned}$$

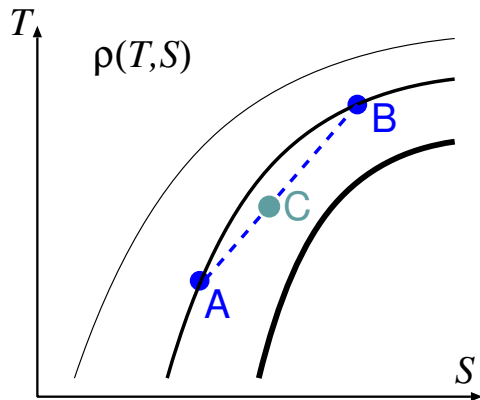
- ▶ cabbeling:  $\sim 7 \times 10^{-6} \text{K}^{-2}$
- ▶ thermobaric:  $\sim -3 \times 10^{-9} \text{K}^{-1} \text{m}^{-2} \text{s}^2$

$S$  による寄与は非線形性が小さい ( $\left(\frac{\partial^2 \rho}{\partial S^2}\right)_0$  などは無視できる)

# cabbling

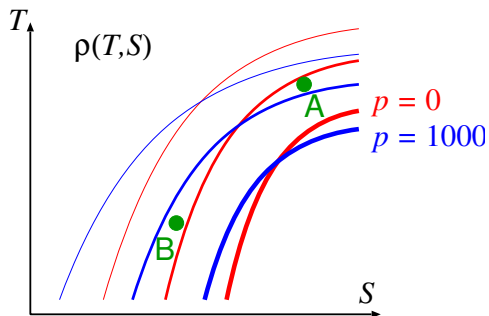
- ▶  $\left(\frac{\partial^2 \rho}{\partial T^2}\right)_0 > 0$
- ▶ 等密度曲線が上に凸
- ▶ 混合すると密度が増加

$$\rho_A = \rho_B < \rho_C$$



# thermobaric

- ▶  $\left(\frac{\partial^2 \rho}{\partial p \partial T}\right)_0 > 0$
- ▶ 深さ (圧力) とともに等密度の曲線が時計周りに回転
- ▶ 圧力を変えると密度の大小関係が逆転
  - ▶  $\rho_A < \rho_B$  @  $p = 0$
  - ▶  $\rho_A > \rho_B$  @  $p = 1000$



# cabbeling

- ▶ 混合→高密度化→下向き流れ
- ▶ もし海洋を全部混ぜてしまうと、海面は 30cm も下がる (Schanze & Schmitt, 2013)

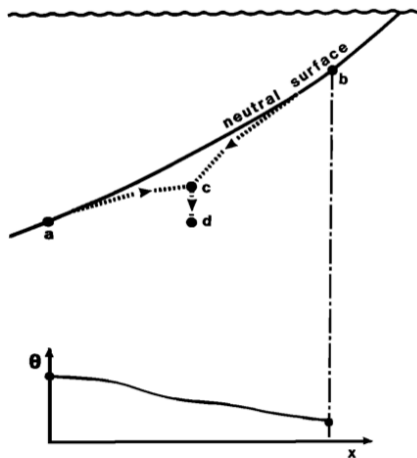


Fig. 5. Figure illustrating the thermobaric vertical advection process (see text for a discussion). Note that the thermobaric effect causes the migration of the two water parcels, *a* and *b*, off the neutral surface as they approach the central point, *c*. Intimate mixing of the two parcels at point *c* consolidates this thermobaric vertical motion and also causes a further sinking of the mixed parcel from *c* to *d*. This further sinking is due to the cabbeling process.

[McDougall, 1987]

# thermobaric convection

Løyring&Weber (1997)

- ▶ 非線形定常解を求めた
  - ▶ 'thermobaric' number

$$\gamma \equiv \frac{g^2 \left( \frac{\partial^2 \rho}{\partial \rho \partial T} \right)_0 \Delta T d^4}{\nu \kappa_T}$$

- ▶ 成層はほぼ中立 (塩分が安定化)

$$\beta \Delta S = \alpha \Delta T$$

- ▶  $\gamma > 3400$  で対流
- ▶ 非線形性 ( $\gamma$ ) が大きくなるとセルが横長に

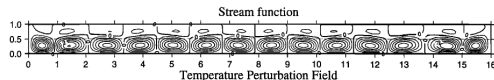


Figure 8. Stationary patterns for the stream function and the perturbation temperature when  $(\alpha_s + \frac{1}{2}\alpha_s d)\Delta T = \beta\Delta S$ . Here  $d = 25$  m and  $\Delta T = 1.0^\circ\text{C}$ , yielding  $\gamma = 1.0 \cdot 10^5$ .

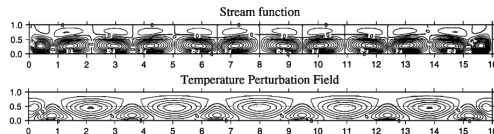


Figure 9. Same as in Figure 8, but now with  $d = 50$  m,  $\Delta T = 0.1^\circ\text{C}$ ,  $\gamma = 1.59 \times 10^5$ .

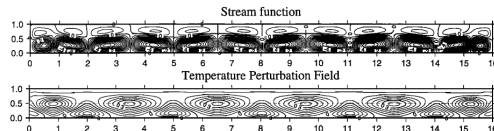
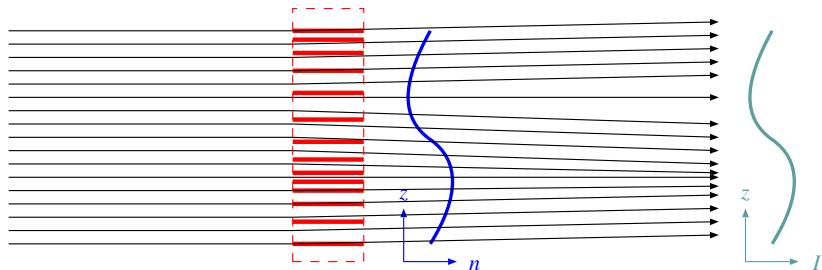


Figure 10. Same as in Figure 8, but now with  $d = 100$  m,  $\Delta T = 0.01^\circ\text{C}$ ,  $\gamma = 2.55 \times 10^5$ .

# シャドウグラフ法



- ▶ 平行光線を入射
- ▶ 密度勾配  $\propto$  屈折率勾配  $\propto$  屈折角度
- ▶ 屈折角度の不均一  $\rightarrow$  光線の収束・発散  $\rightarrow$  明度  $I$

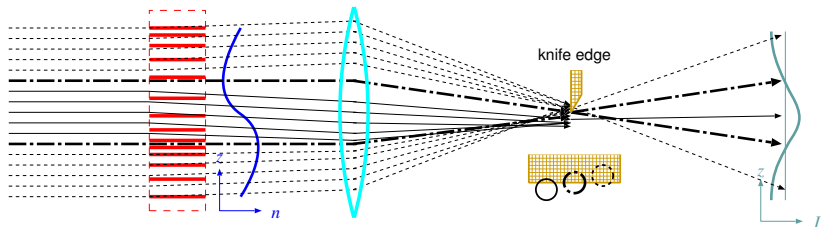
$\Rightarrow$  密度の2階微分が明るさとして可視化される

$$I \propto \frac{\partial}{\partial z} \frac{\partial n}{\partial z} \propto \frac{\partial^2 \rho}{\partial z^2}$$

光が透過する2次元面に拡張して、

$$I \propto \nabla^2 \rho$$

# シュリーレン法



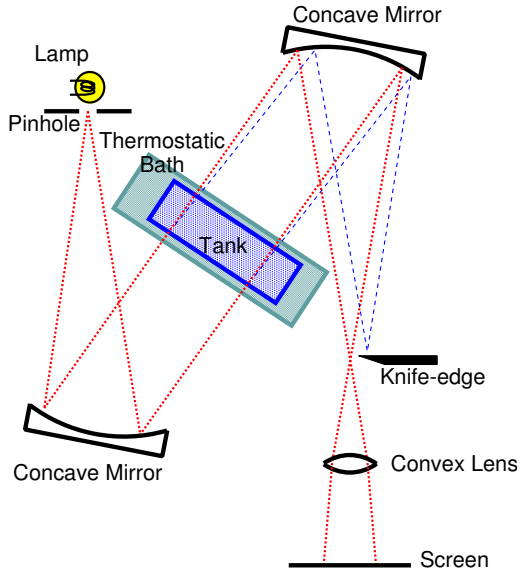
- ▶ 平行光線を入射
- ▶ 密度勾配  $\propto$  屈折率勾配  $\propto$  屈折角度
- ▶ レンズ  $\rightarrow$  焦点面に光源の像  $\rightarrow$  ‘ナイフエッジ’ で部分的に遮光
- ▶ 光源像の位置の変化  $\rightarrow$  遮ぎられる割合の変化  $\rightarrow$  明度  $I$

- ▶ 密度の1階微分が明るさとして可視化される
- ▶ 方向性がある ( $k$ : ナイフの法線)

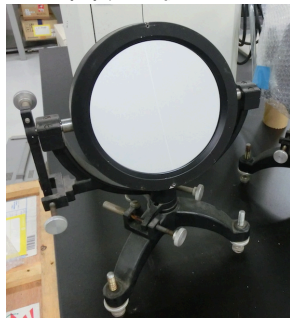
$$I \propto k \cdot \nabla \rho$$



# 実際のシュリーレン撮影



実験室では長い光路が  
取れないことが多いので、  
凸レンズの代わりに凹面鏡を使う



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